The Perceptron

LING 282/482: Deep Learning for Computational Linguistics

C.M. Downey

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Supervised Learning Basics

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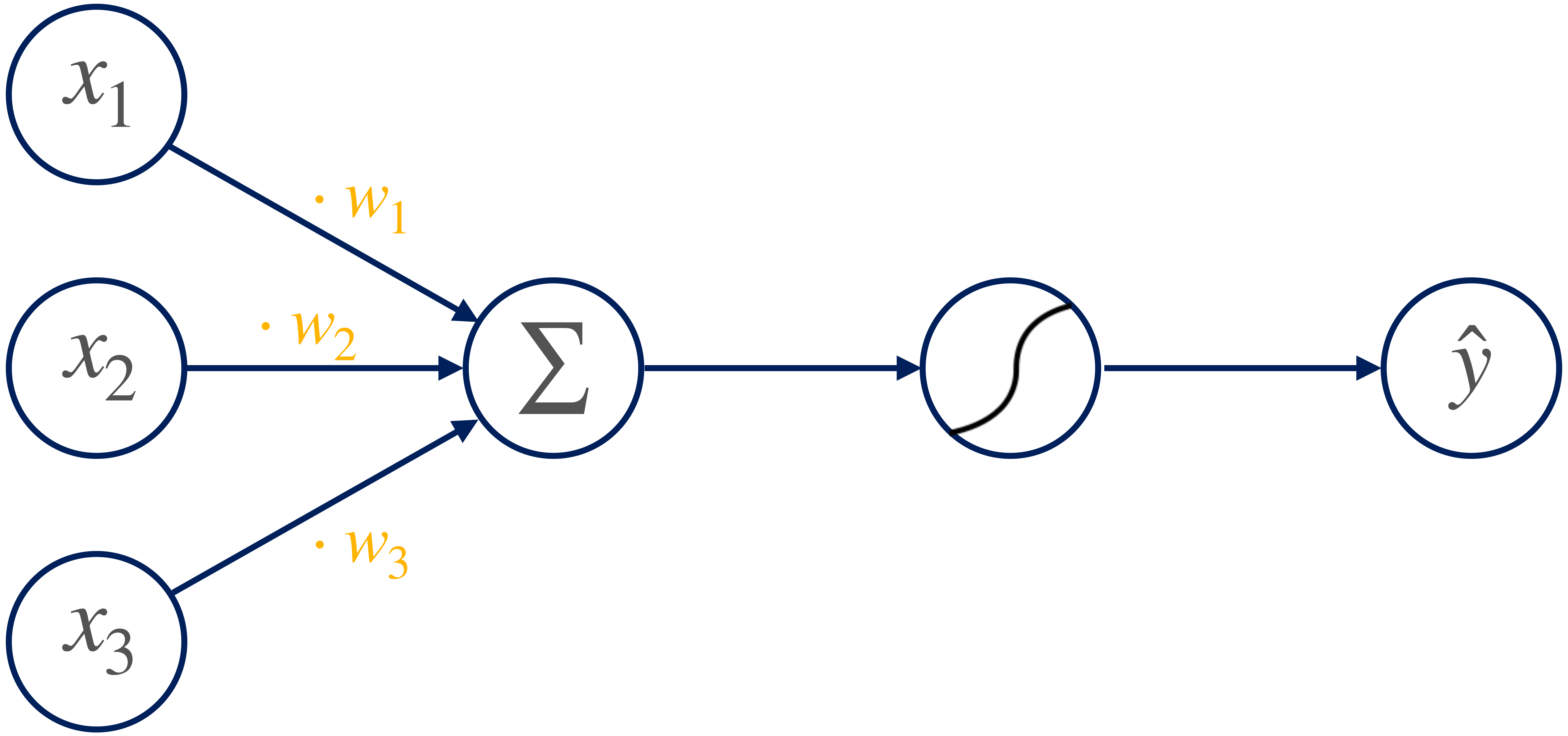
    - {(30, False), (33, False), (35, False), (37, True), (39, True)}
- Goal: learn the function that **best matches the dataset**

Perceptron

- Last lecture: **vectors, matrices, and linear transformations**
  - How do these relate to Neural Networks?
- We'll demonstrate using the **simplest Neural Network**: the **Perceptron**



Perceptron Components

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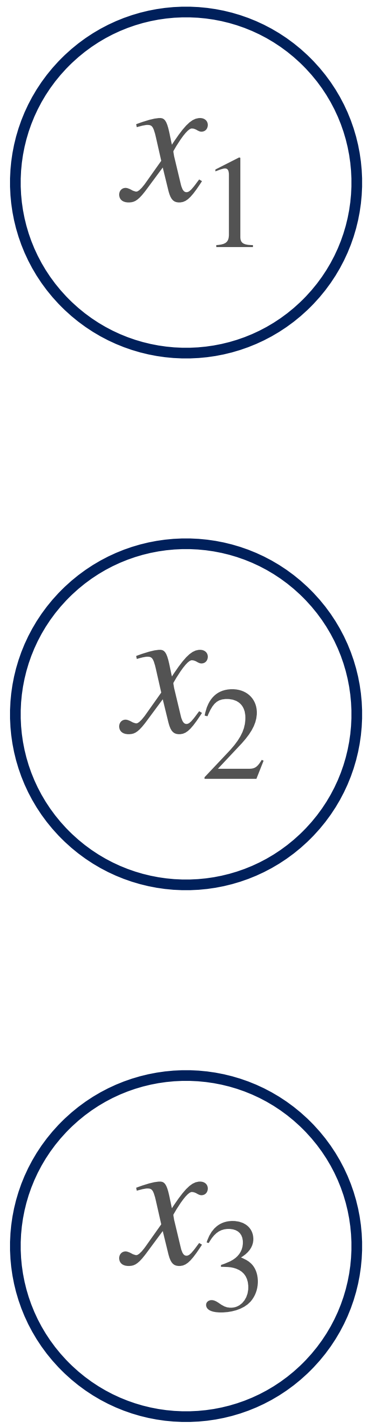

inputs x
(numbers)

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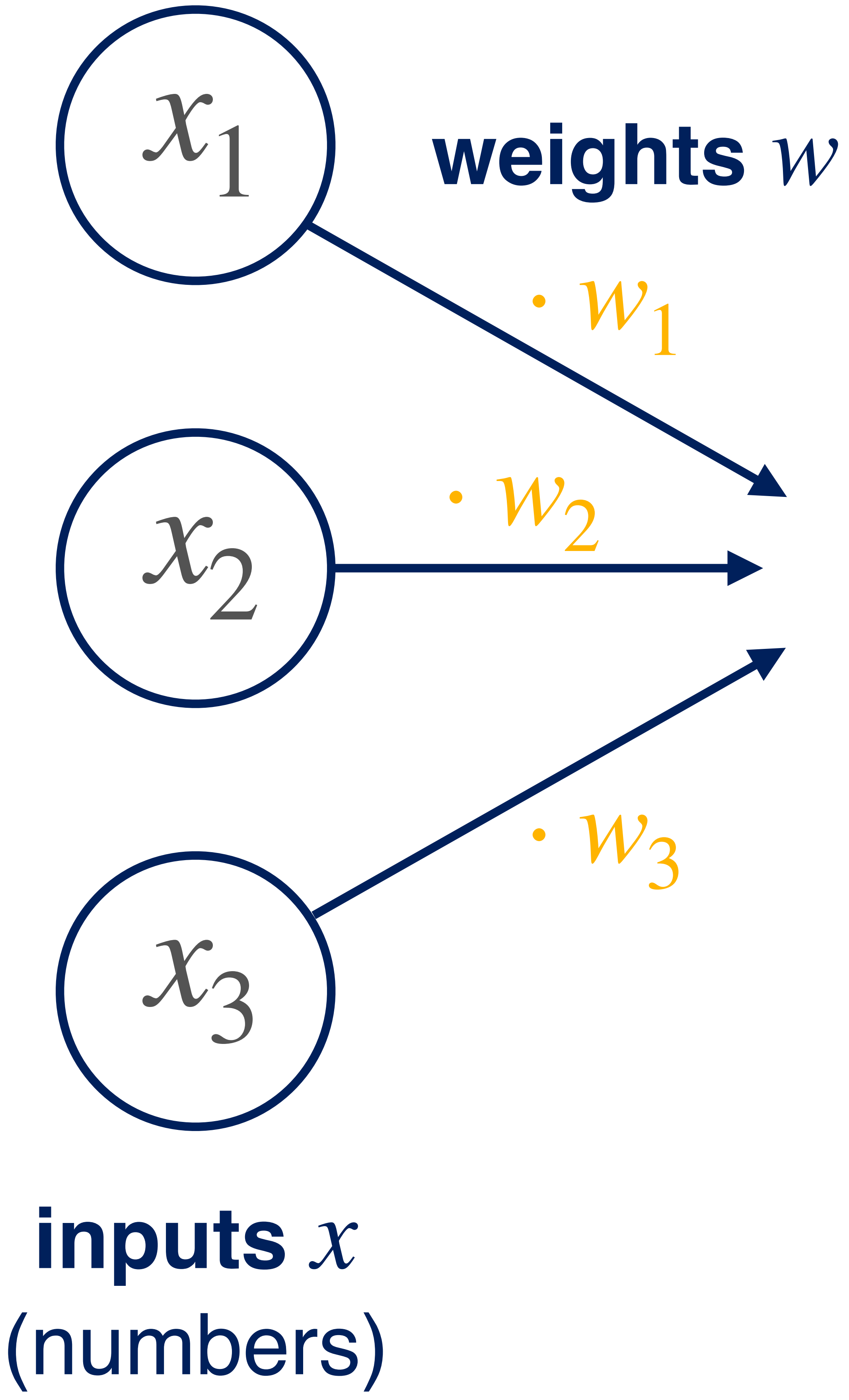

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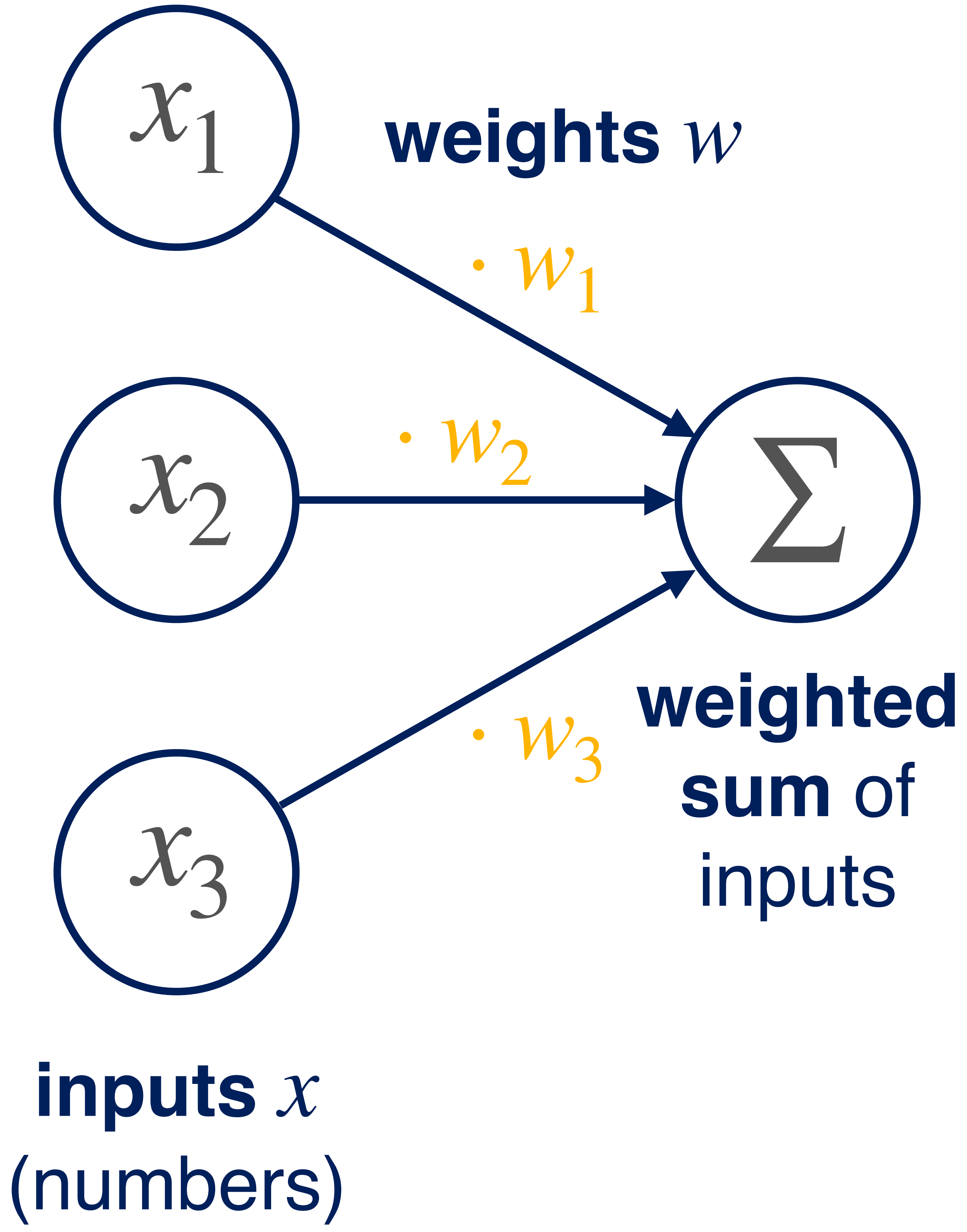

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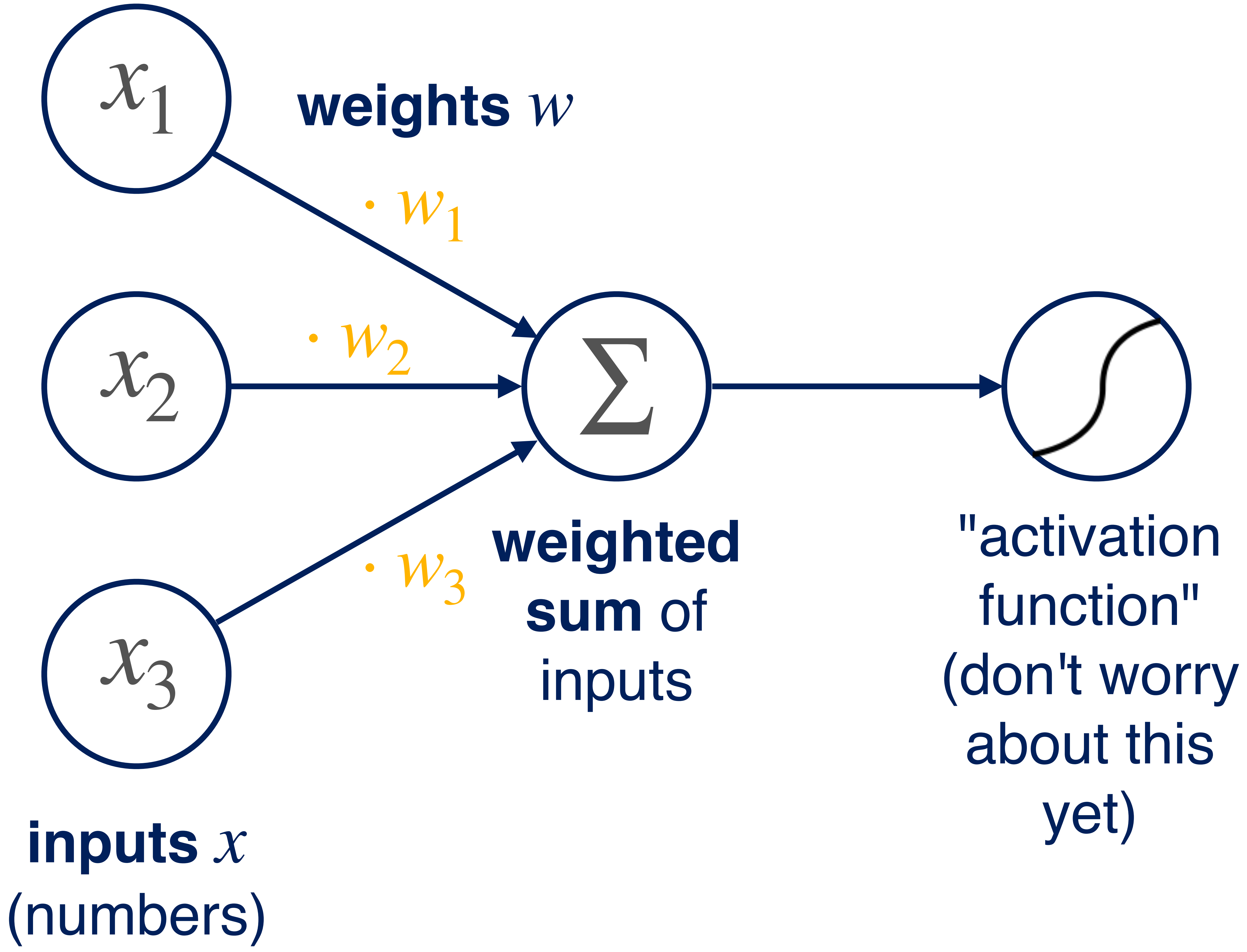

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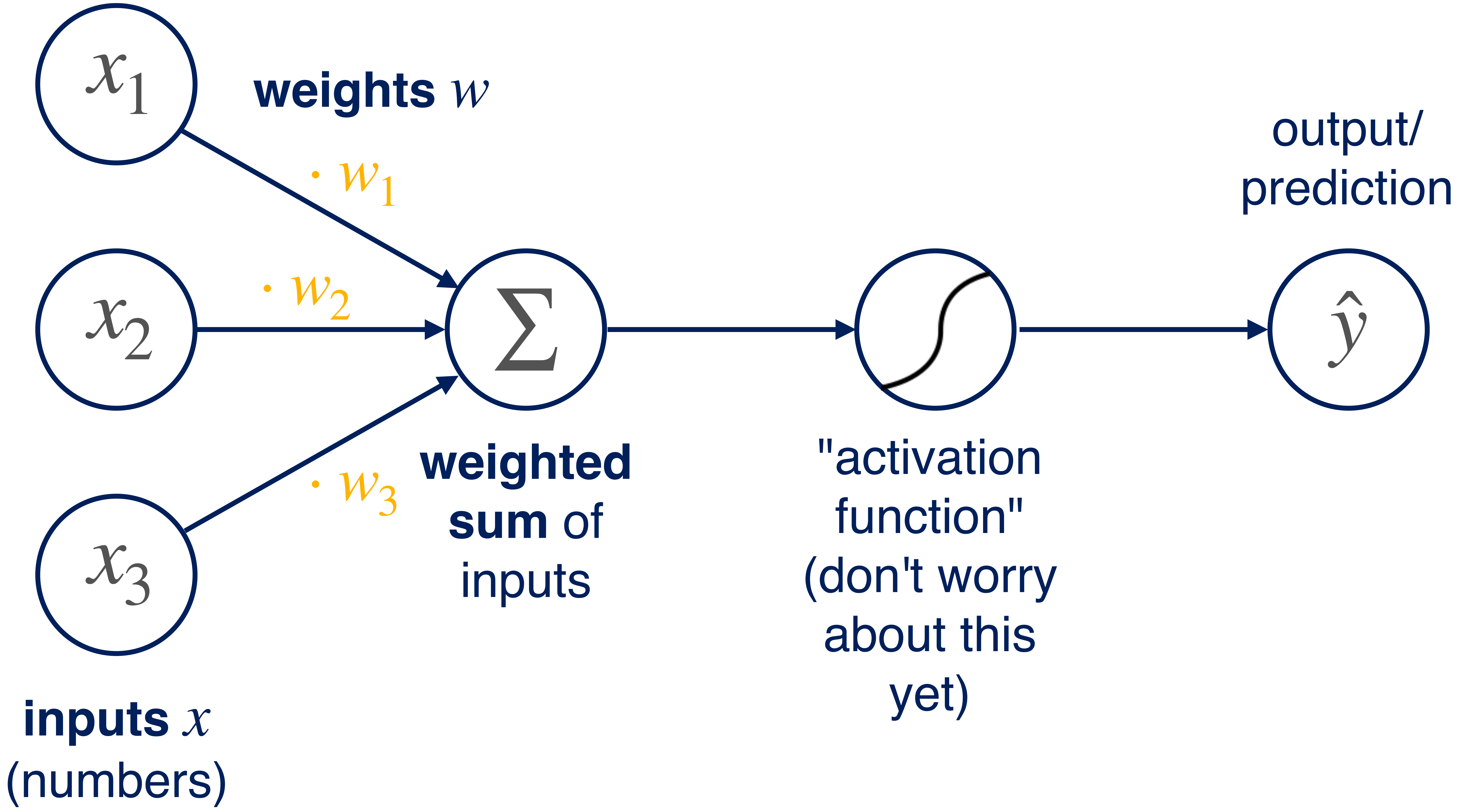

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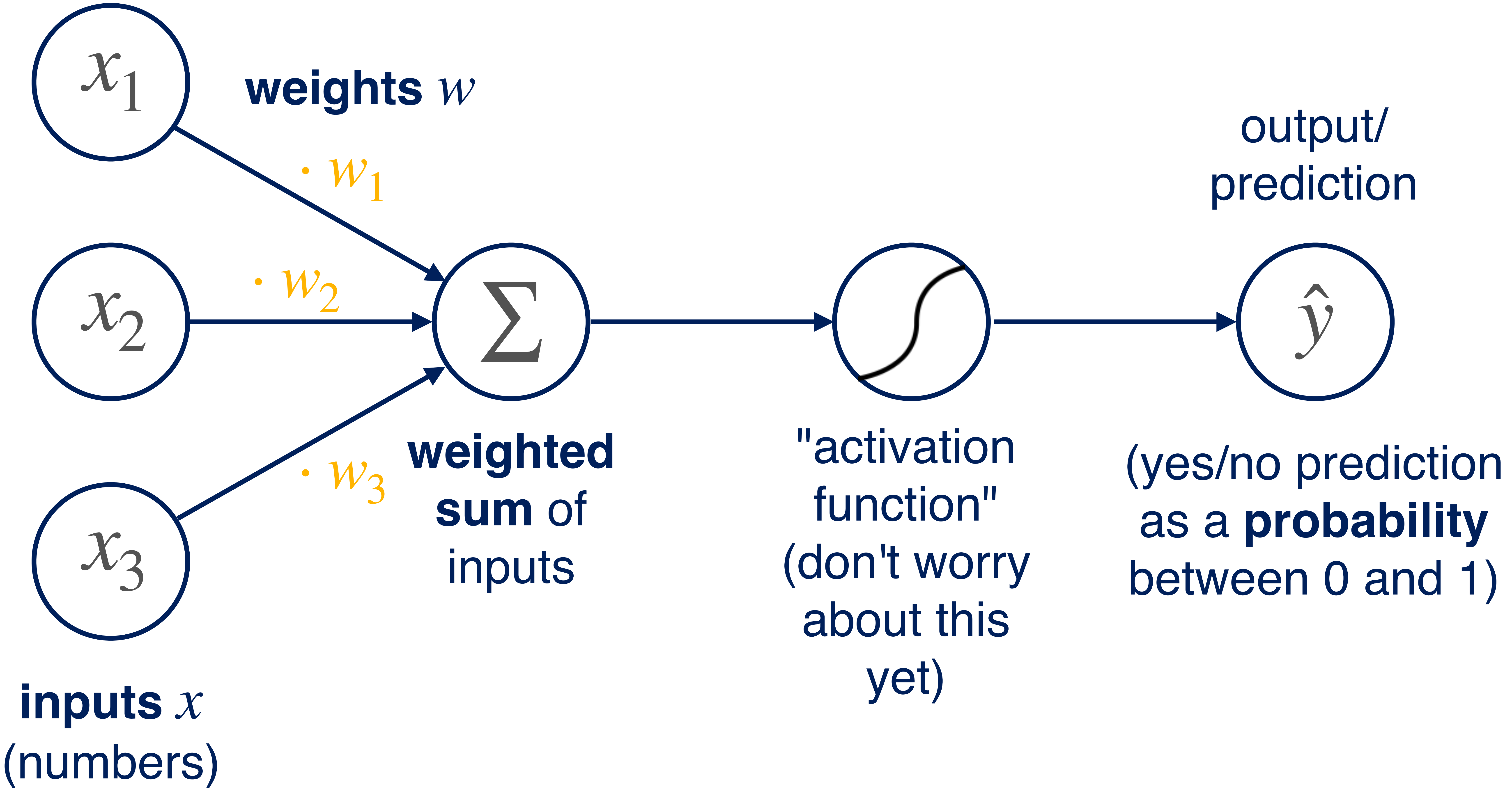

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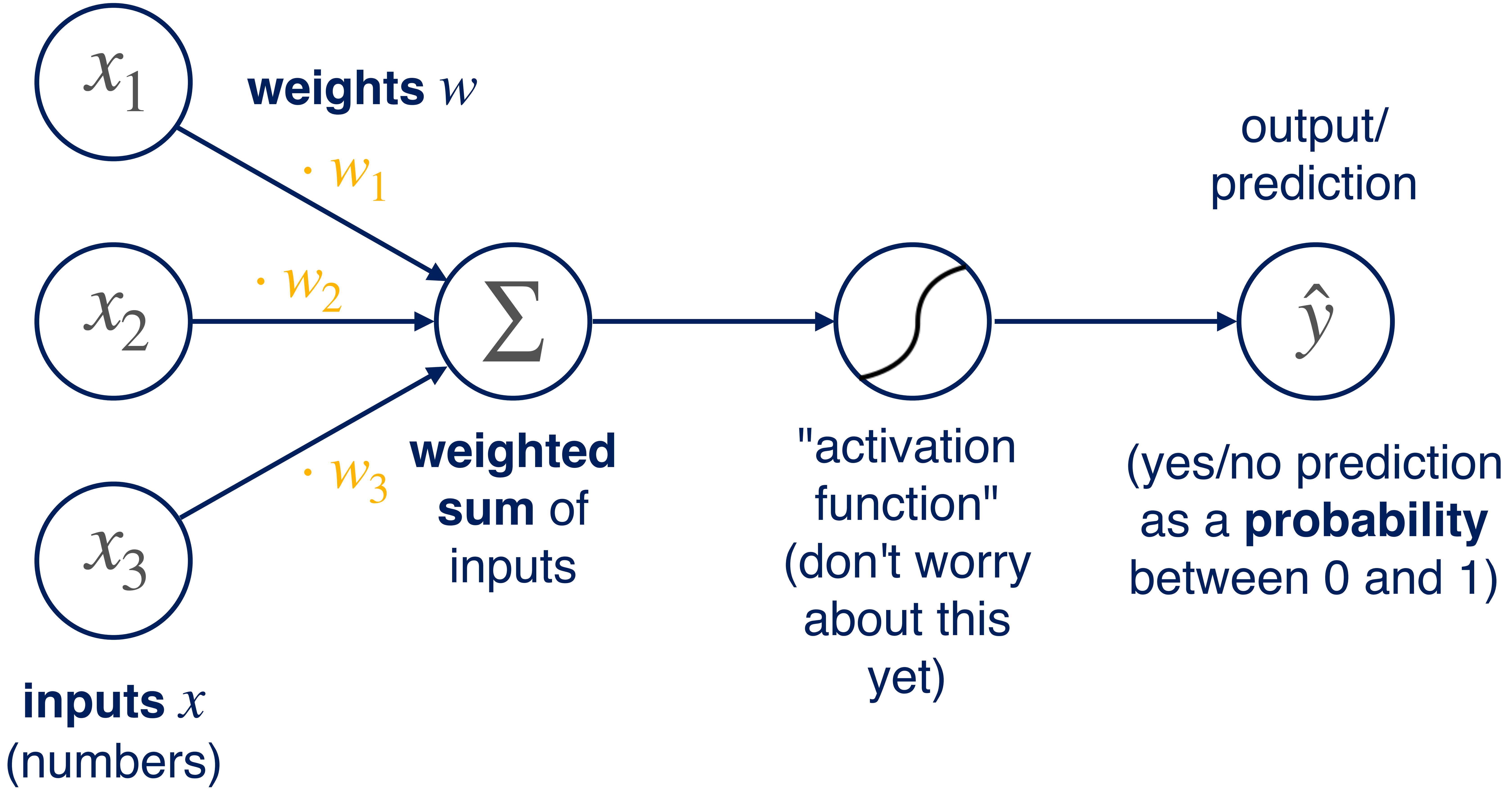

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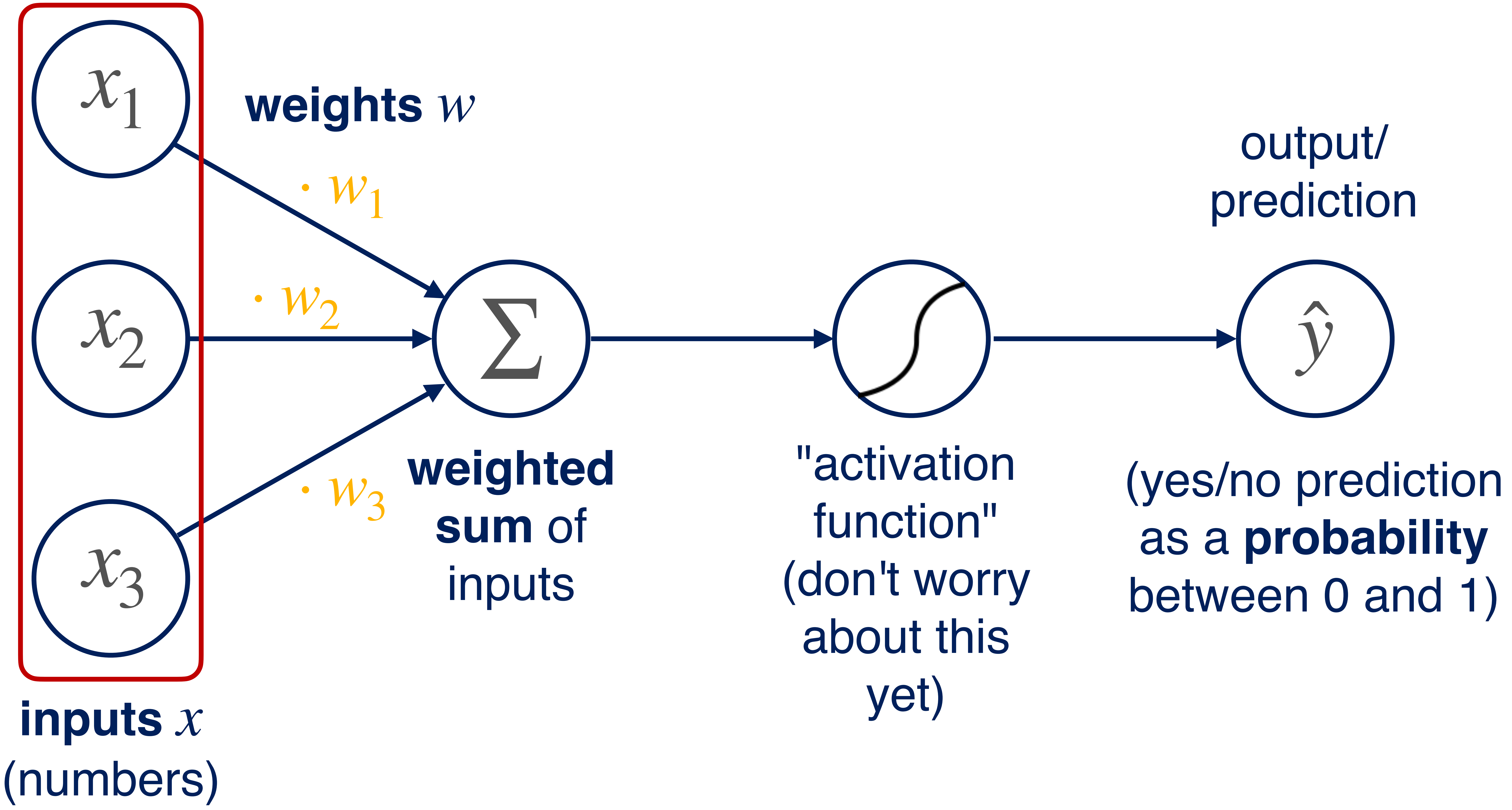

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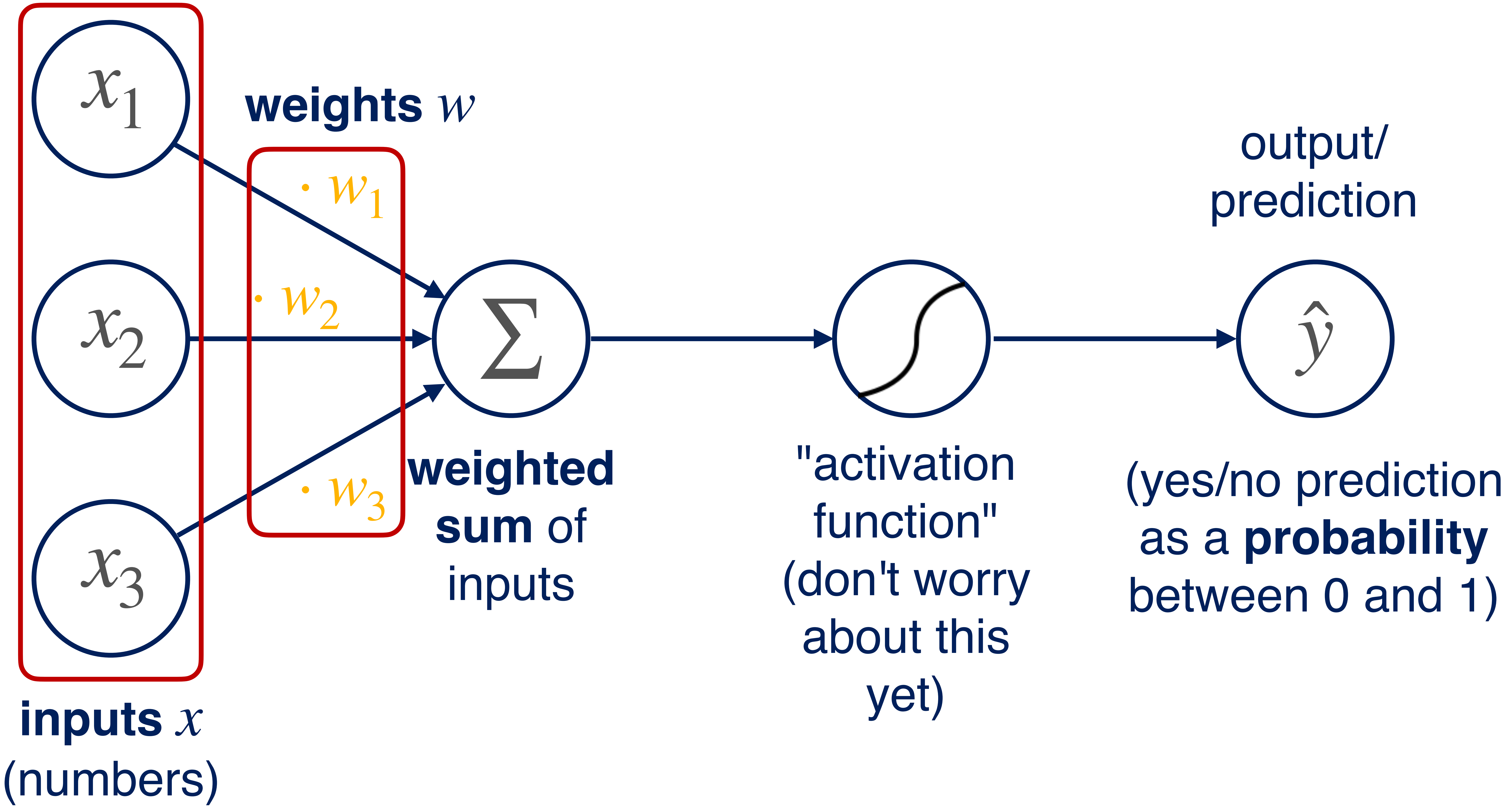
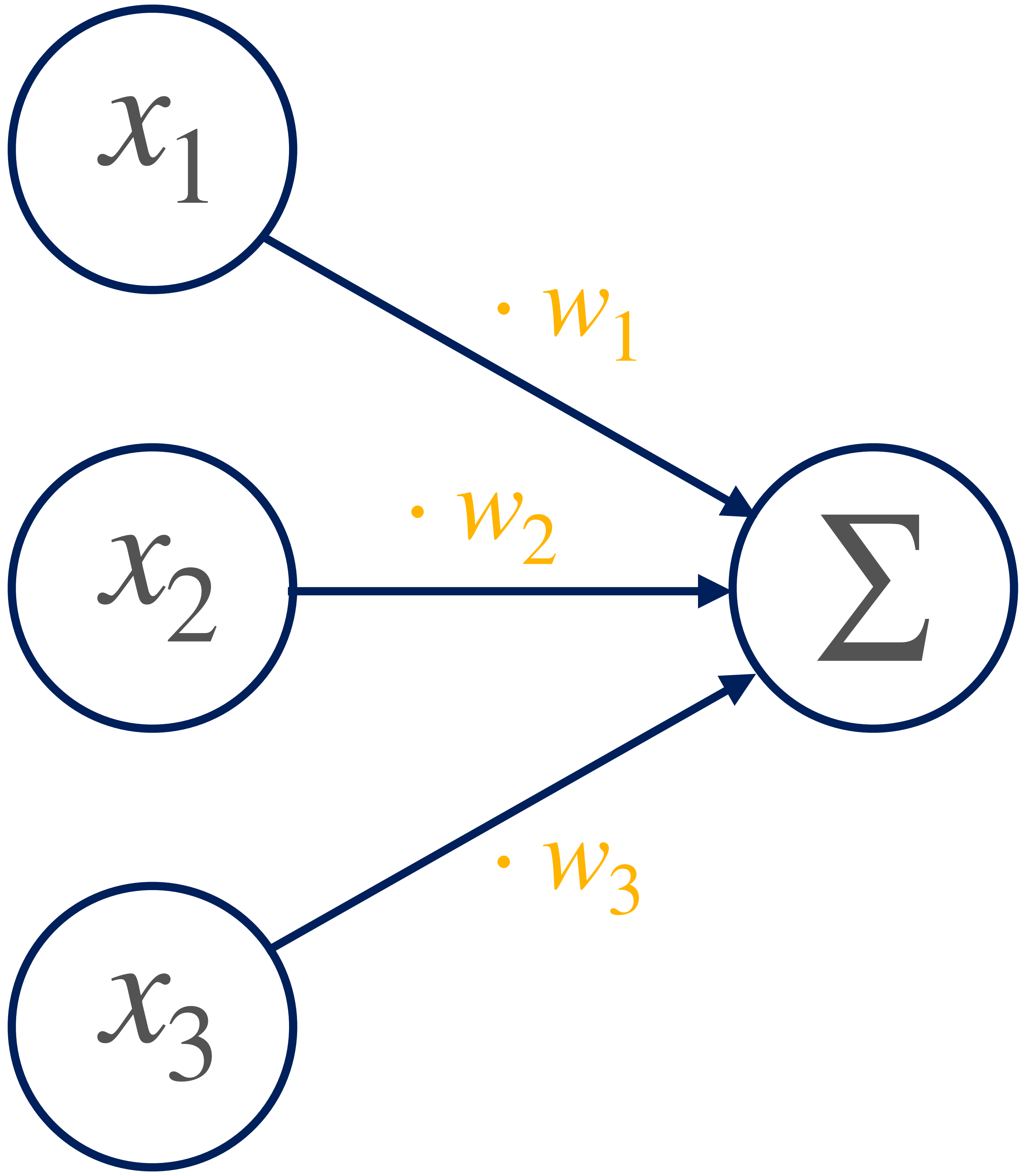


Vectors in the Perceptron



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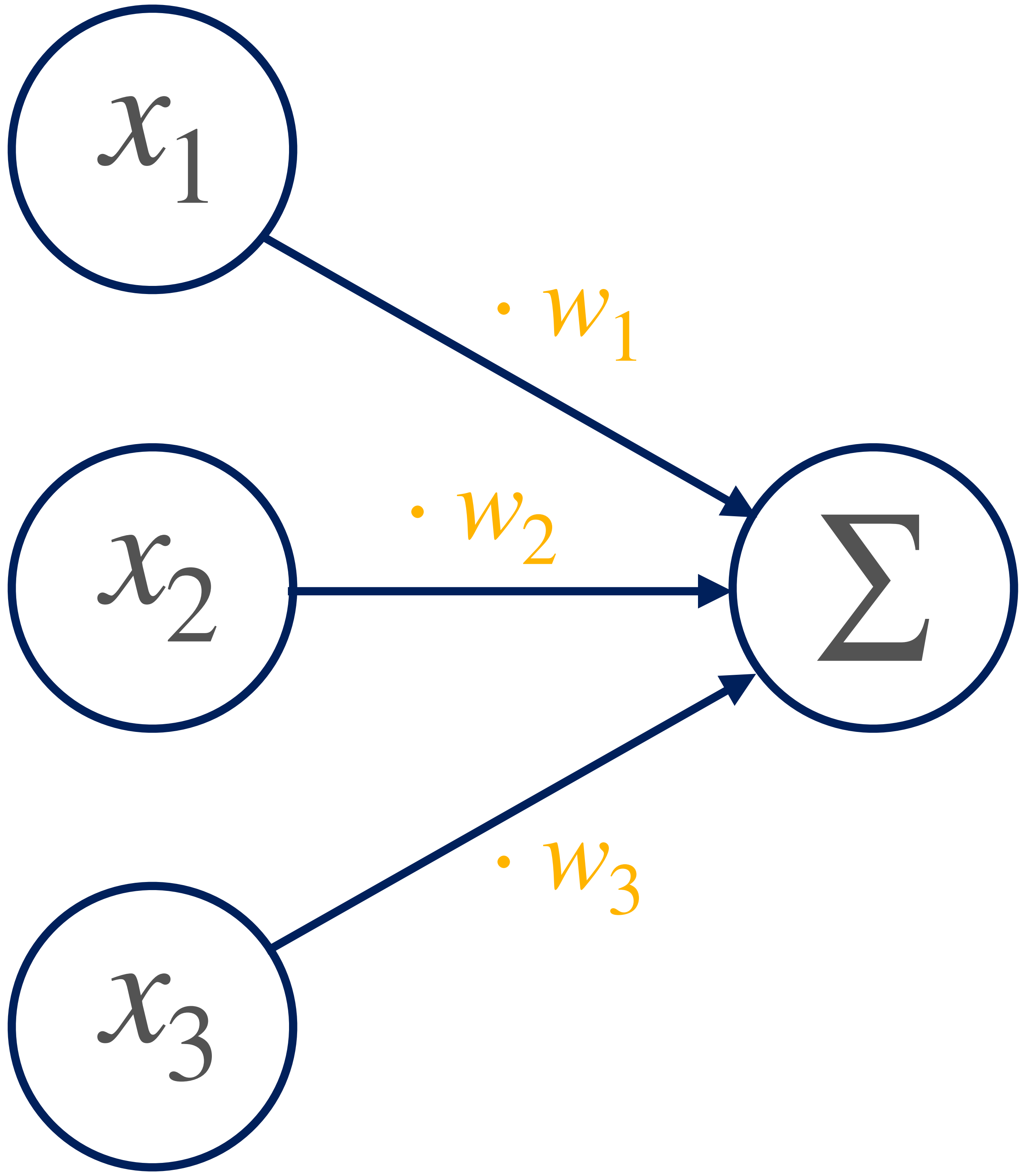

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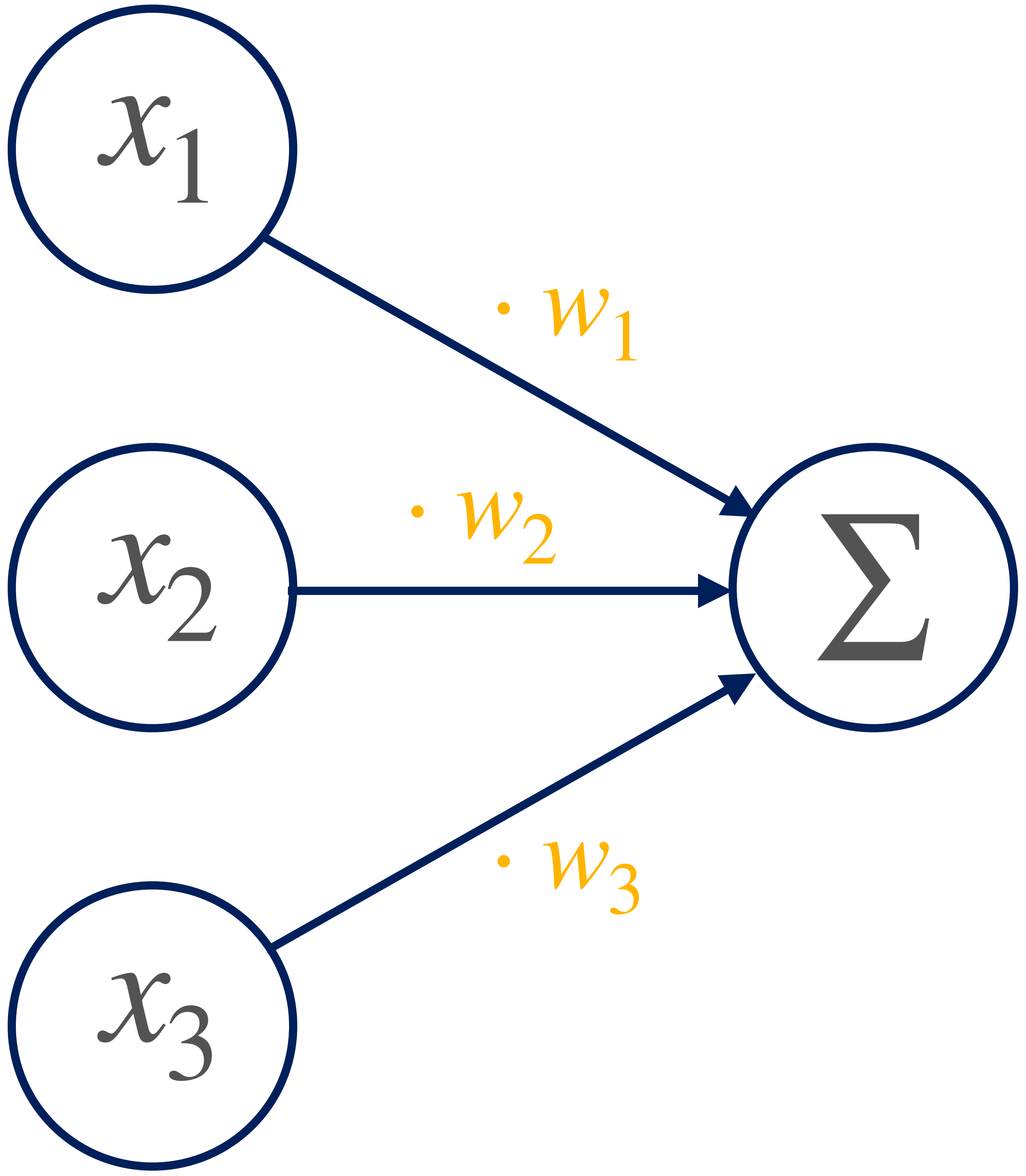

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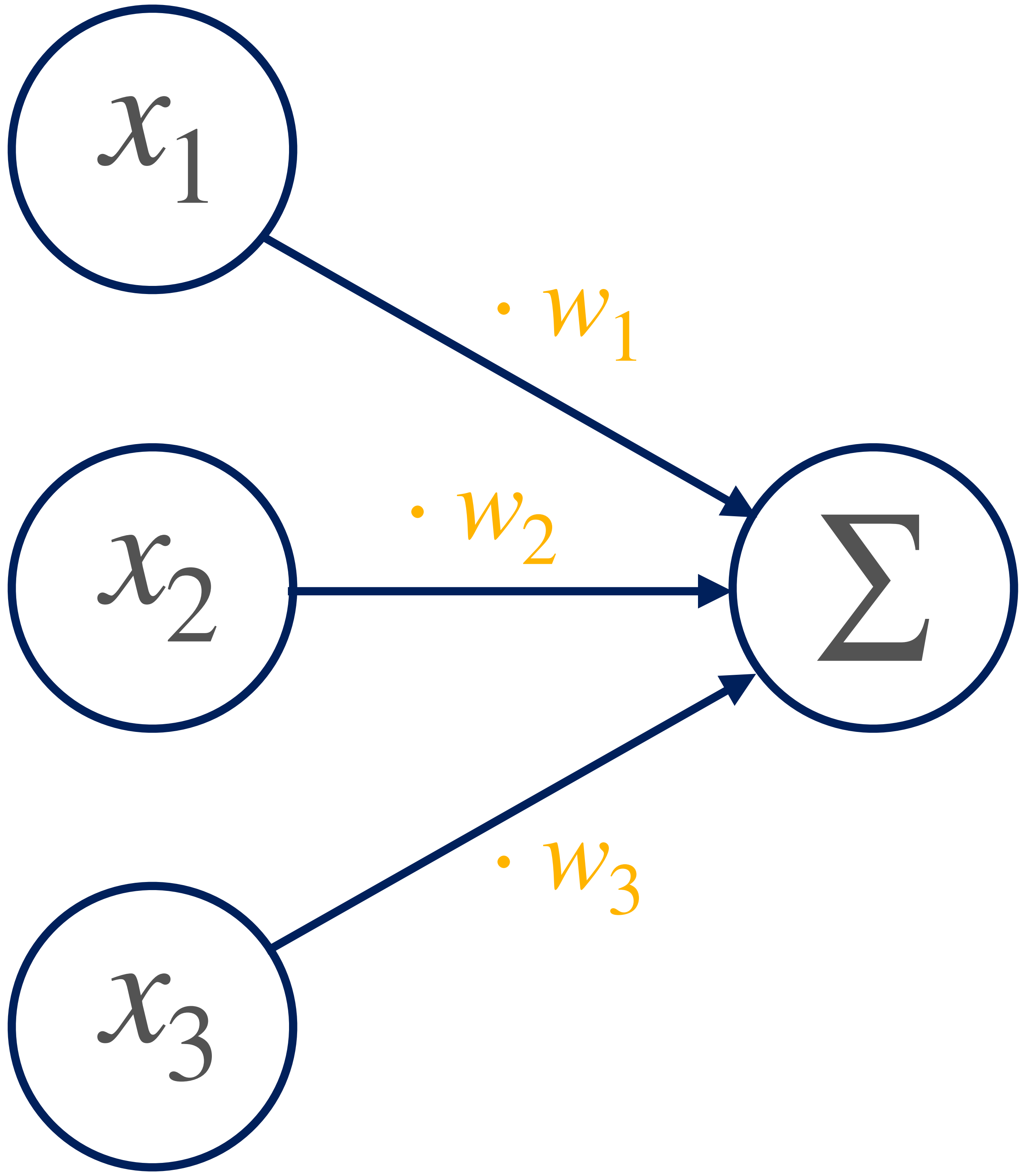

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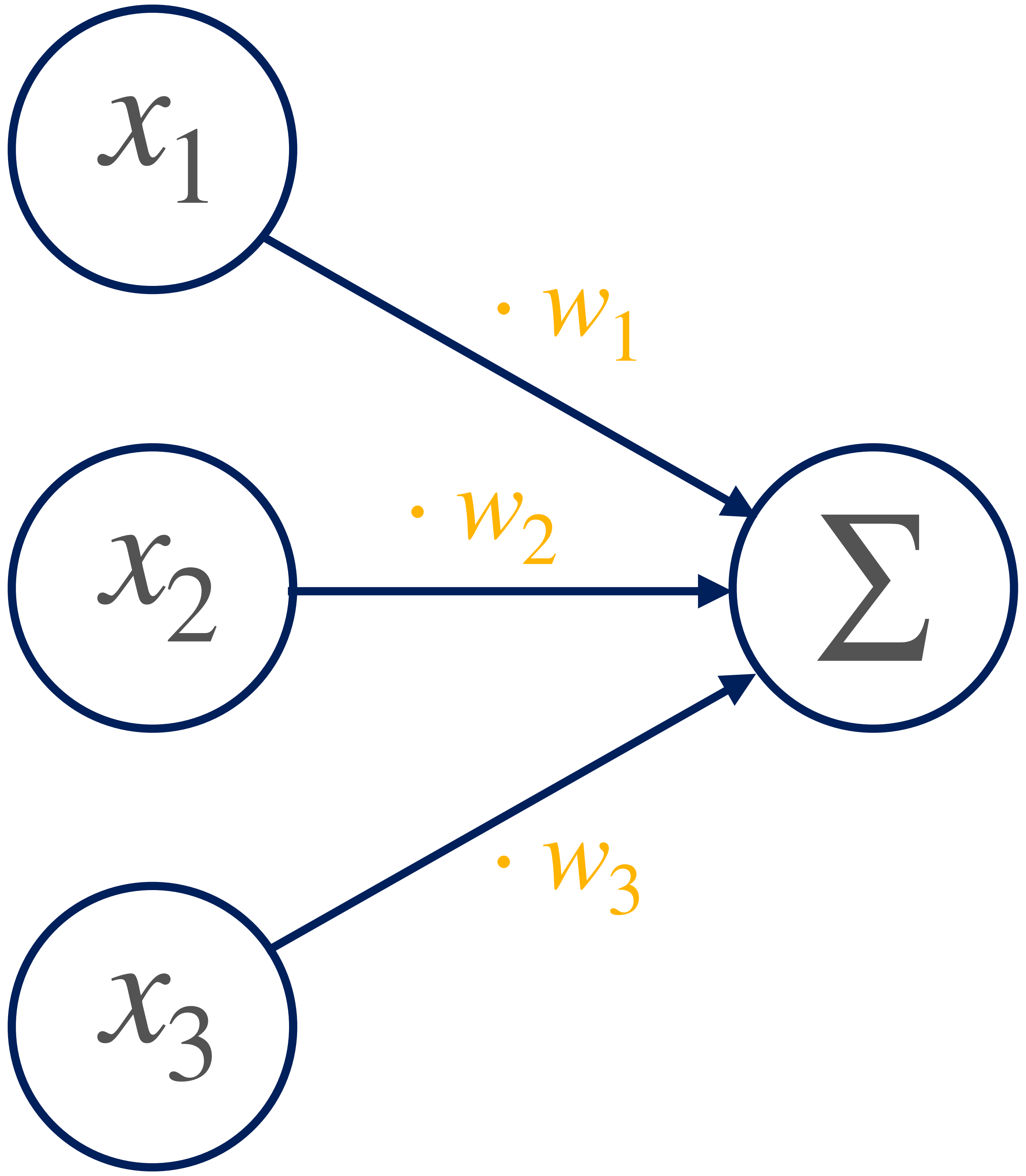

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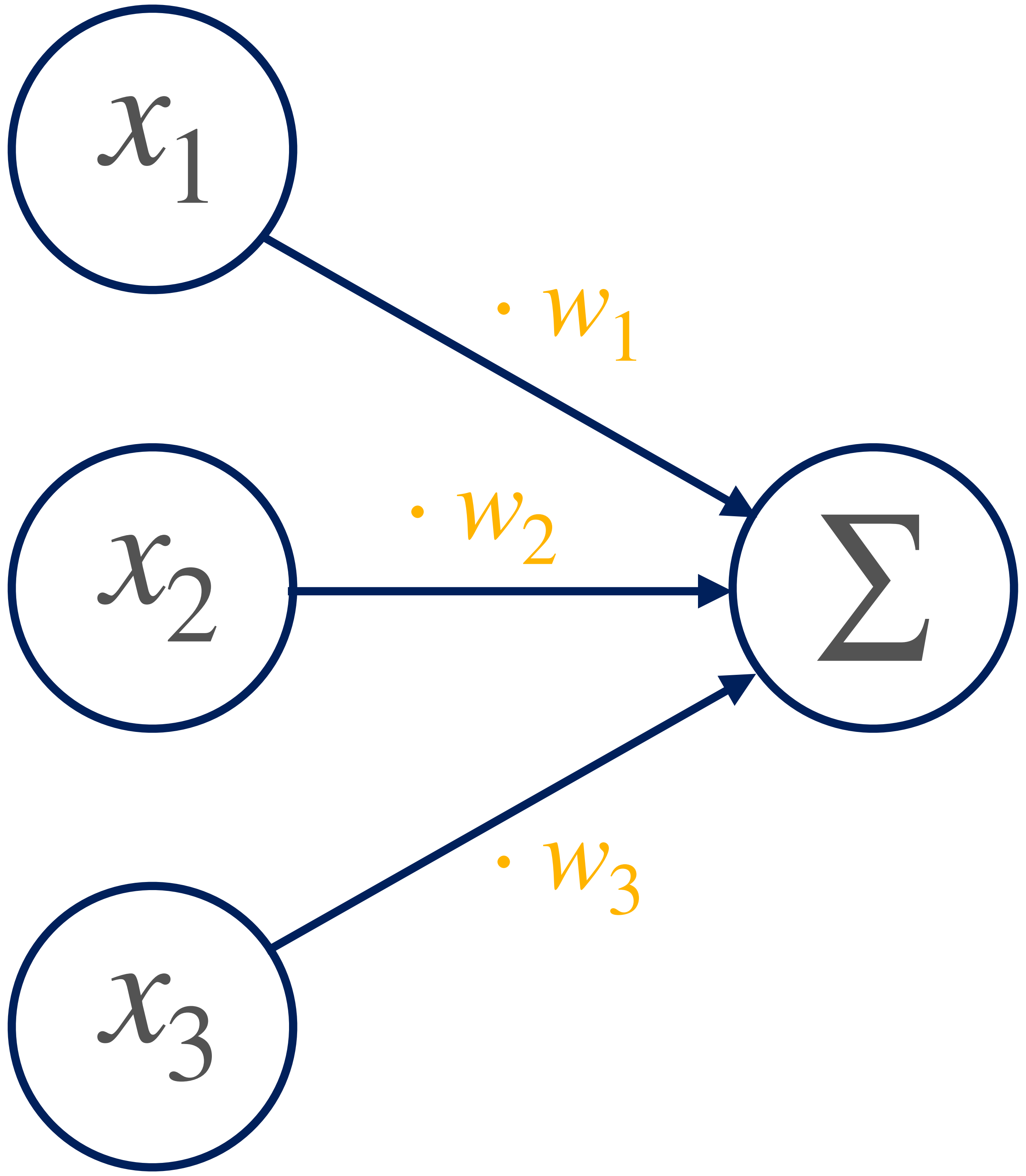

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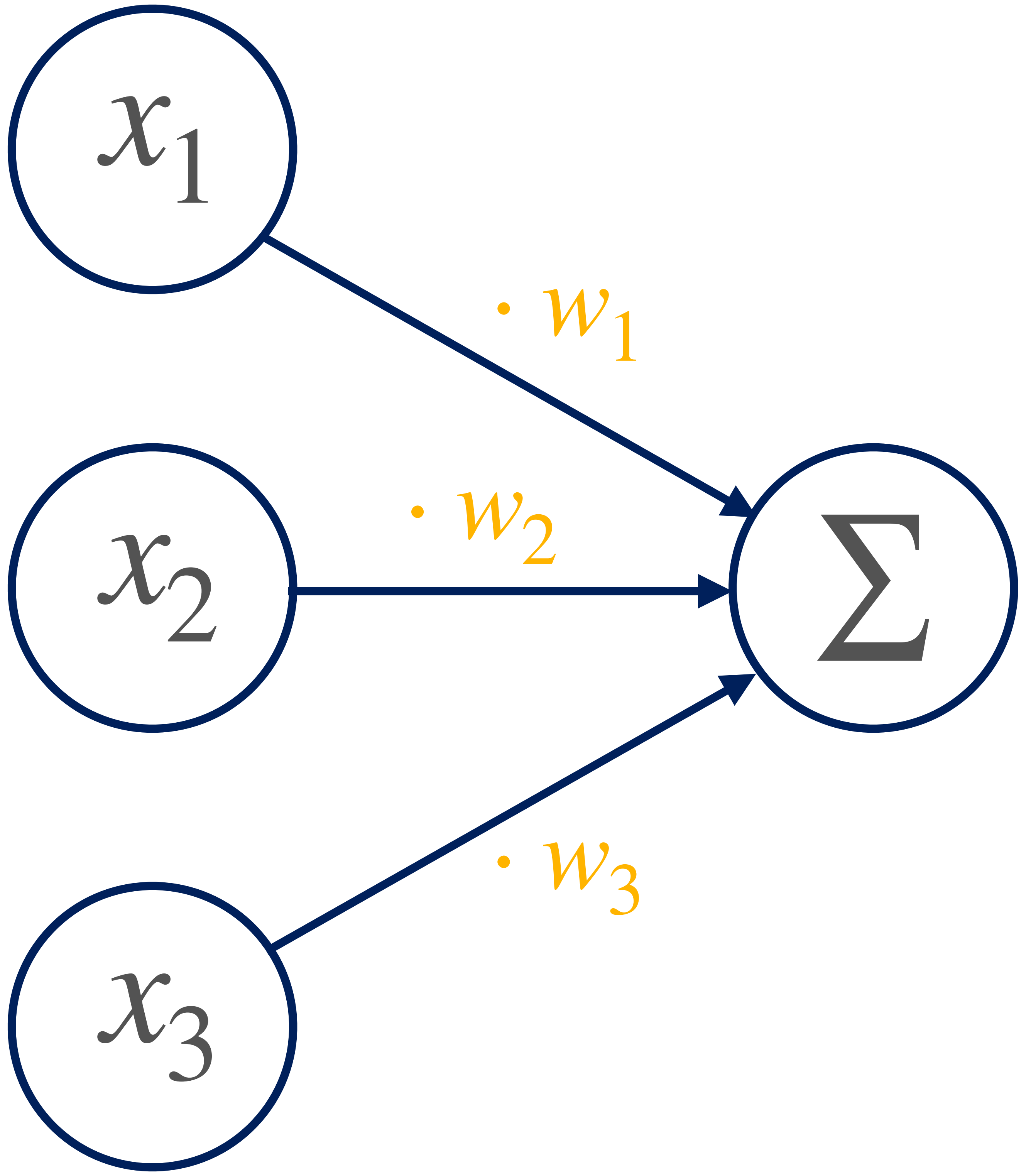

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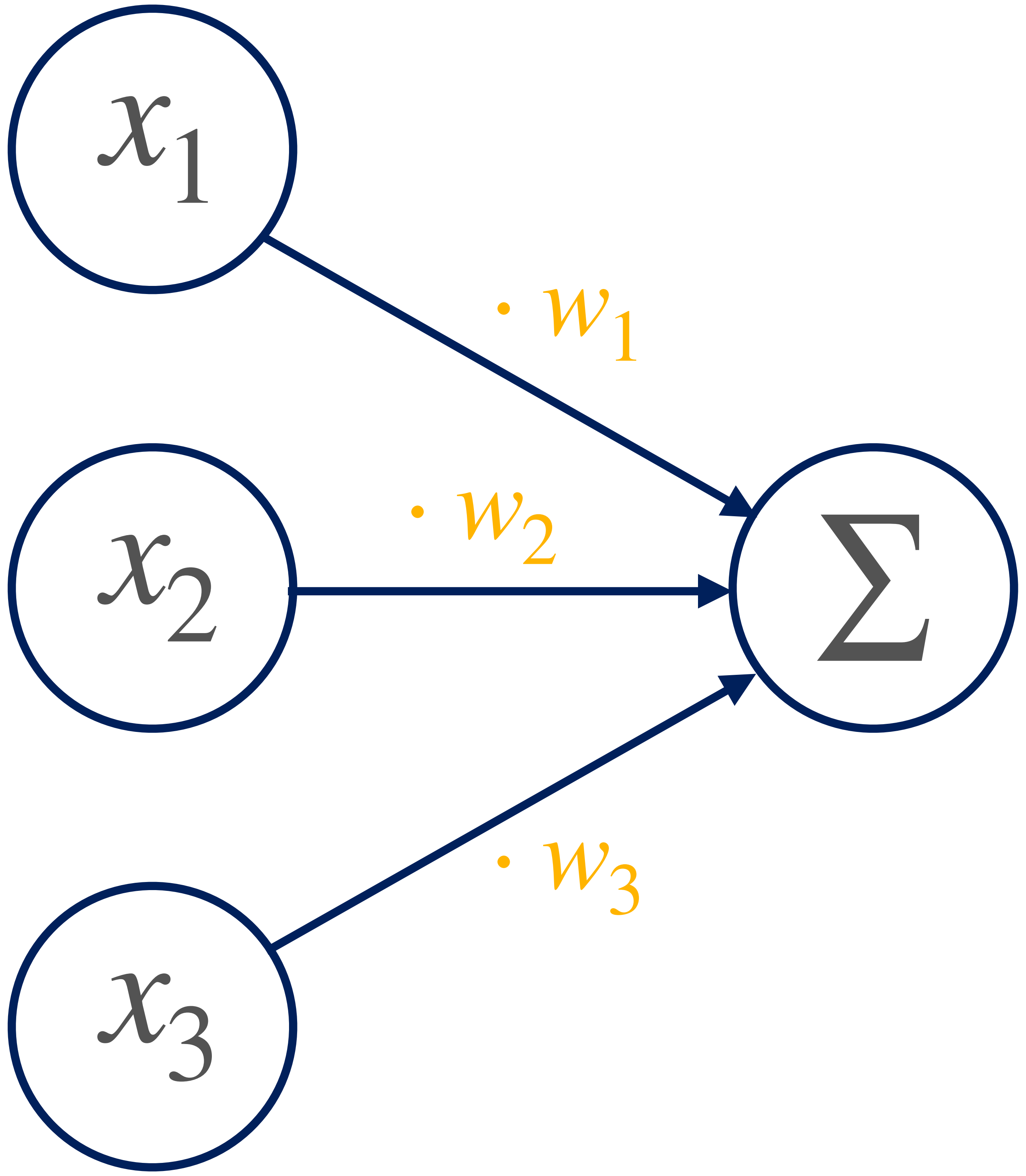

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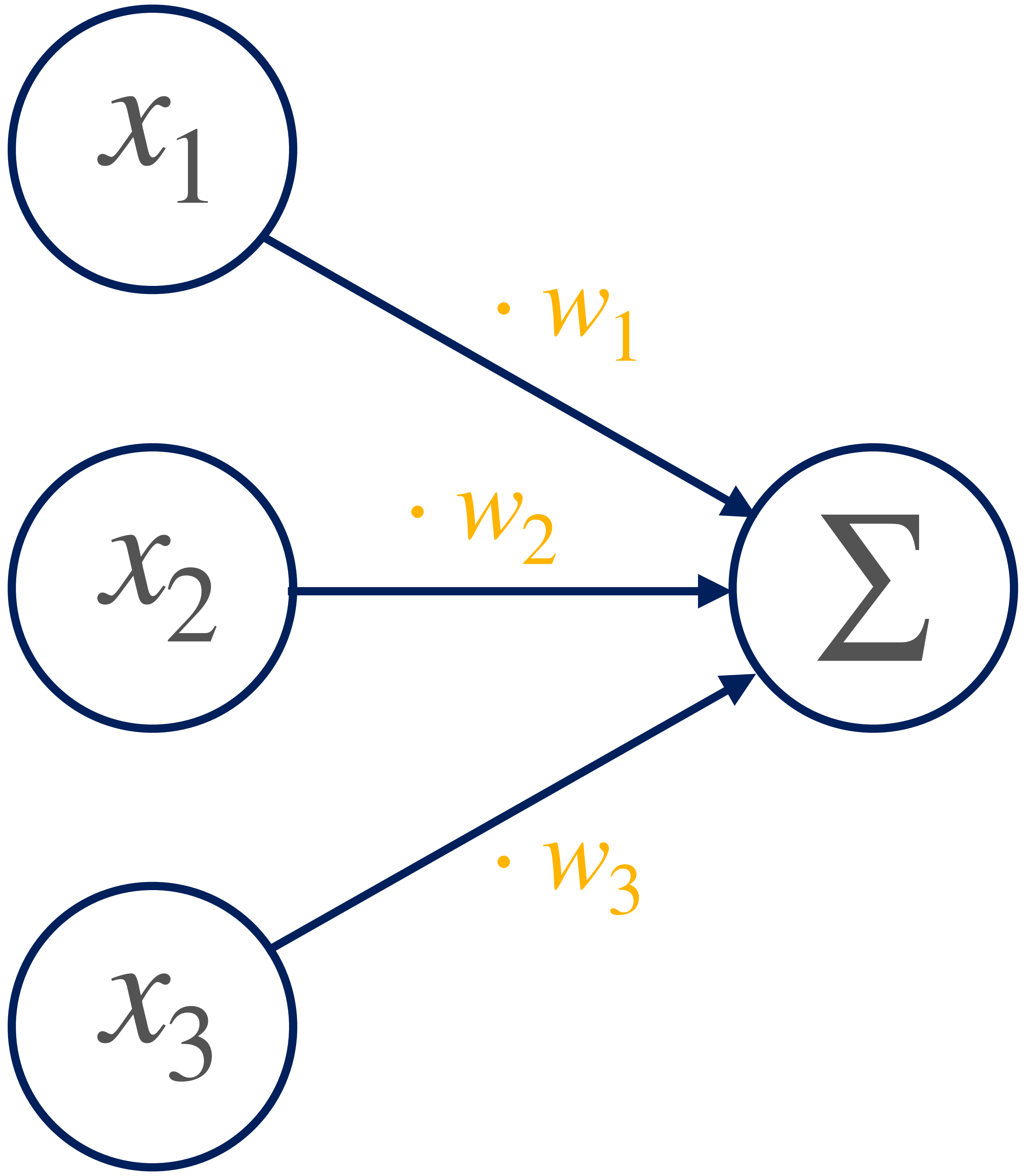

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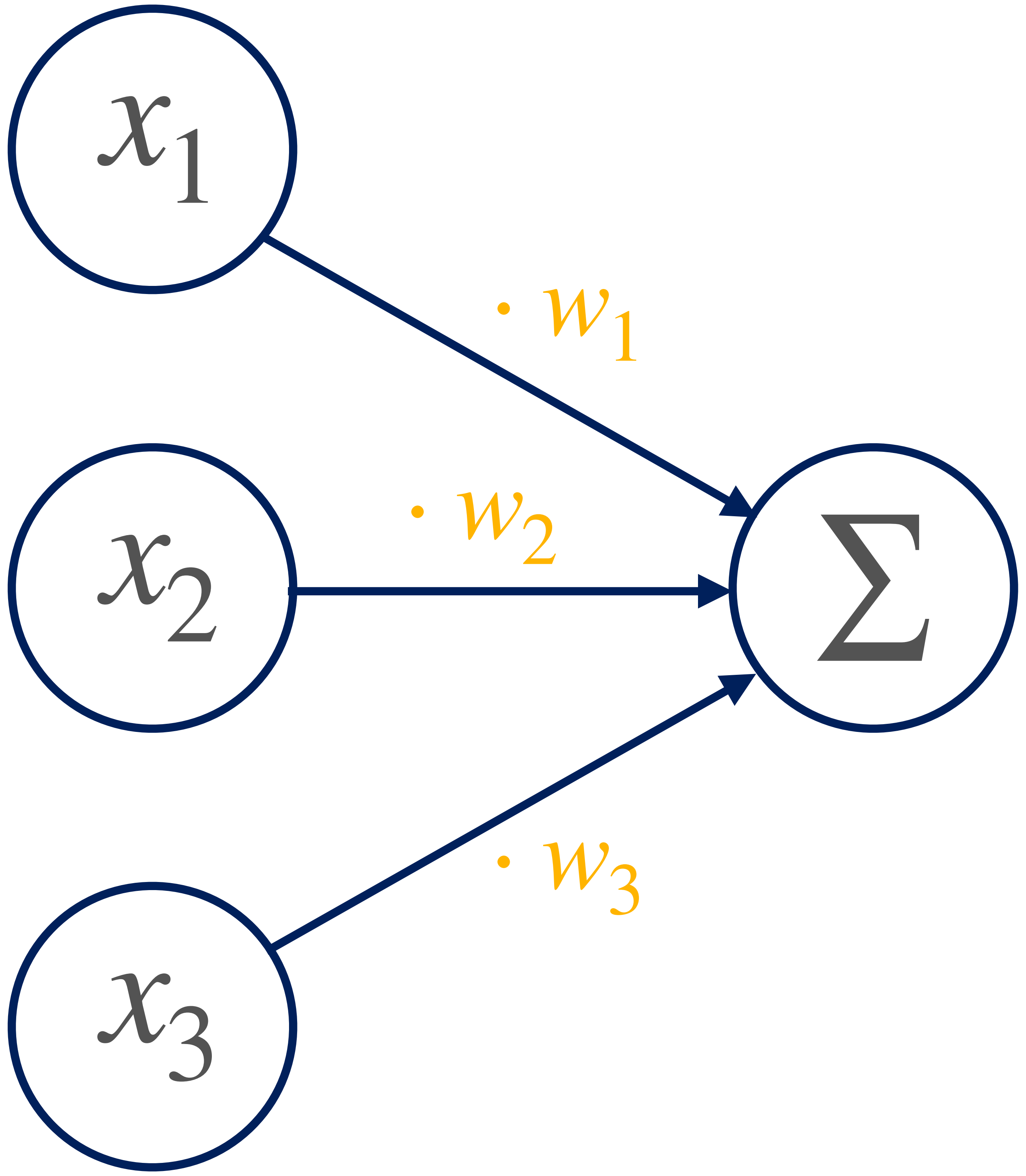
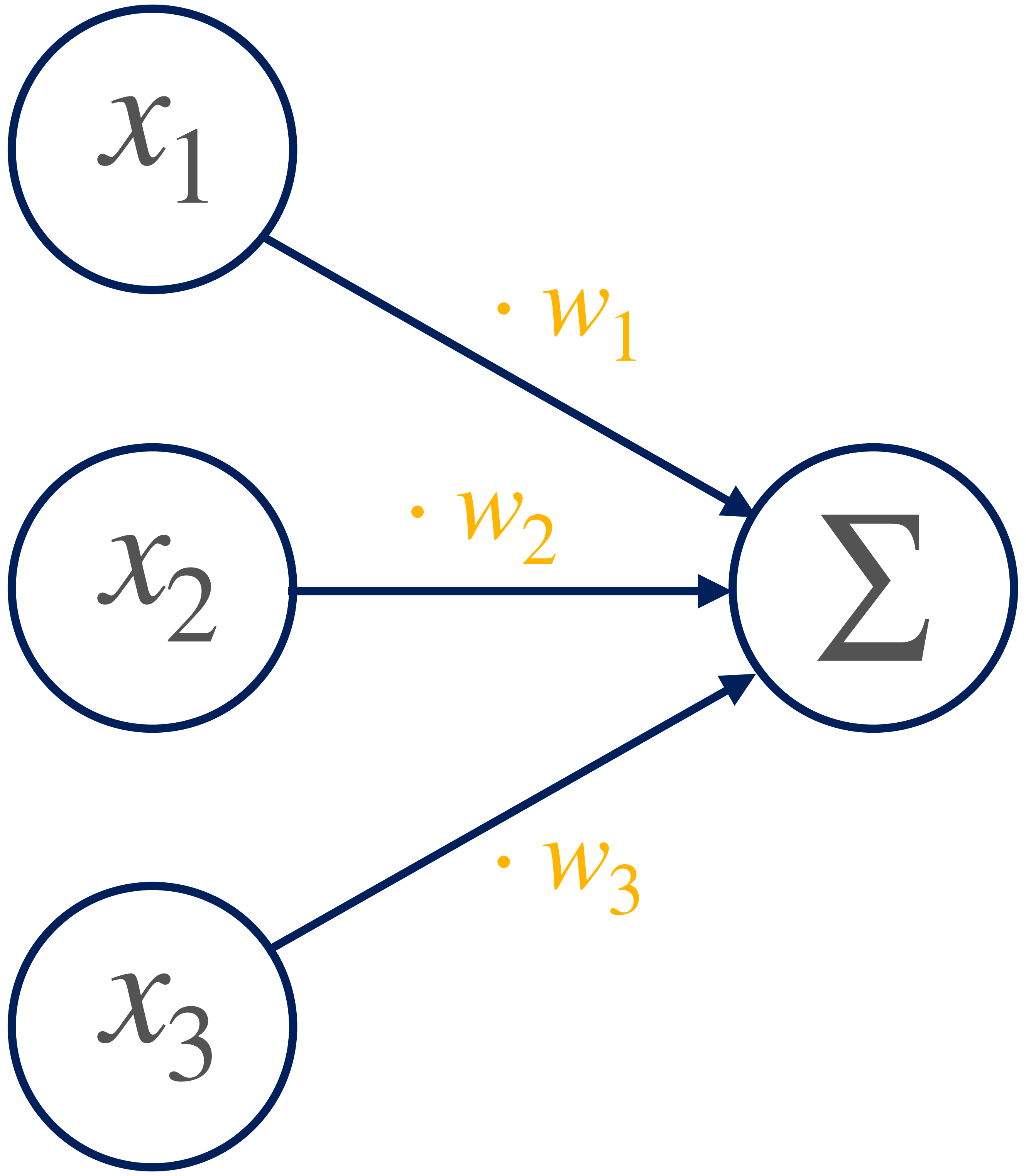
  - (where σ is the activation function)



Perceptron Weights



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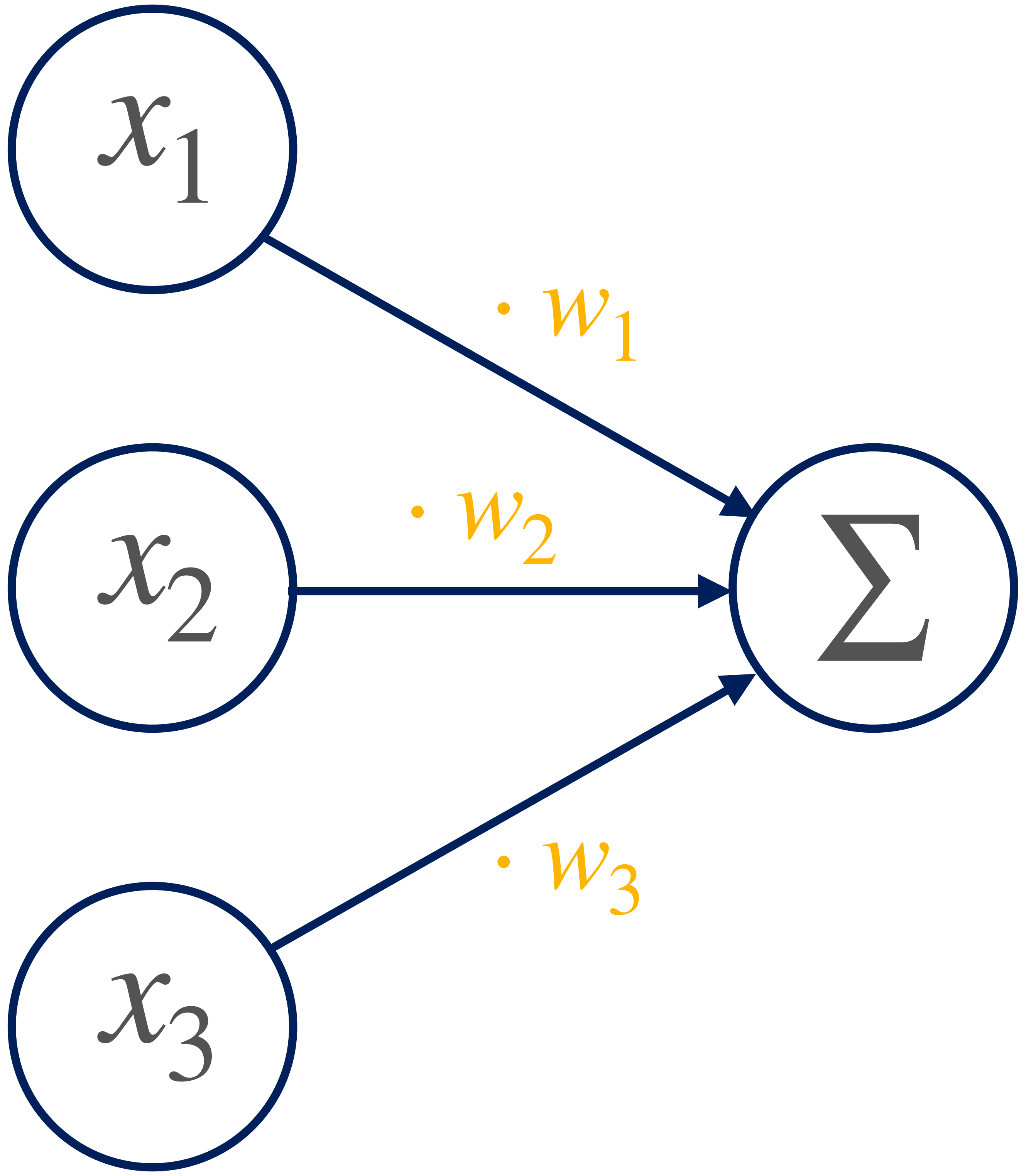

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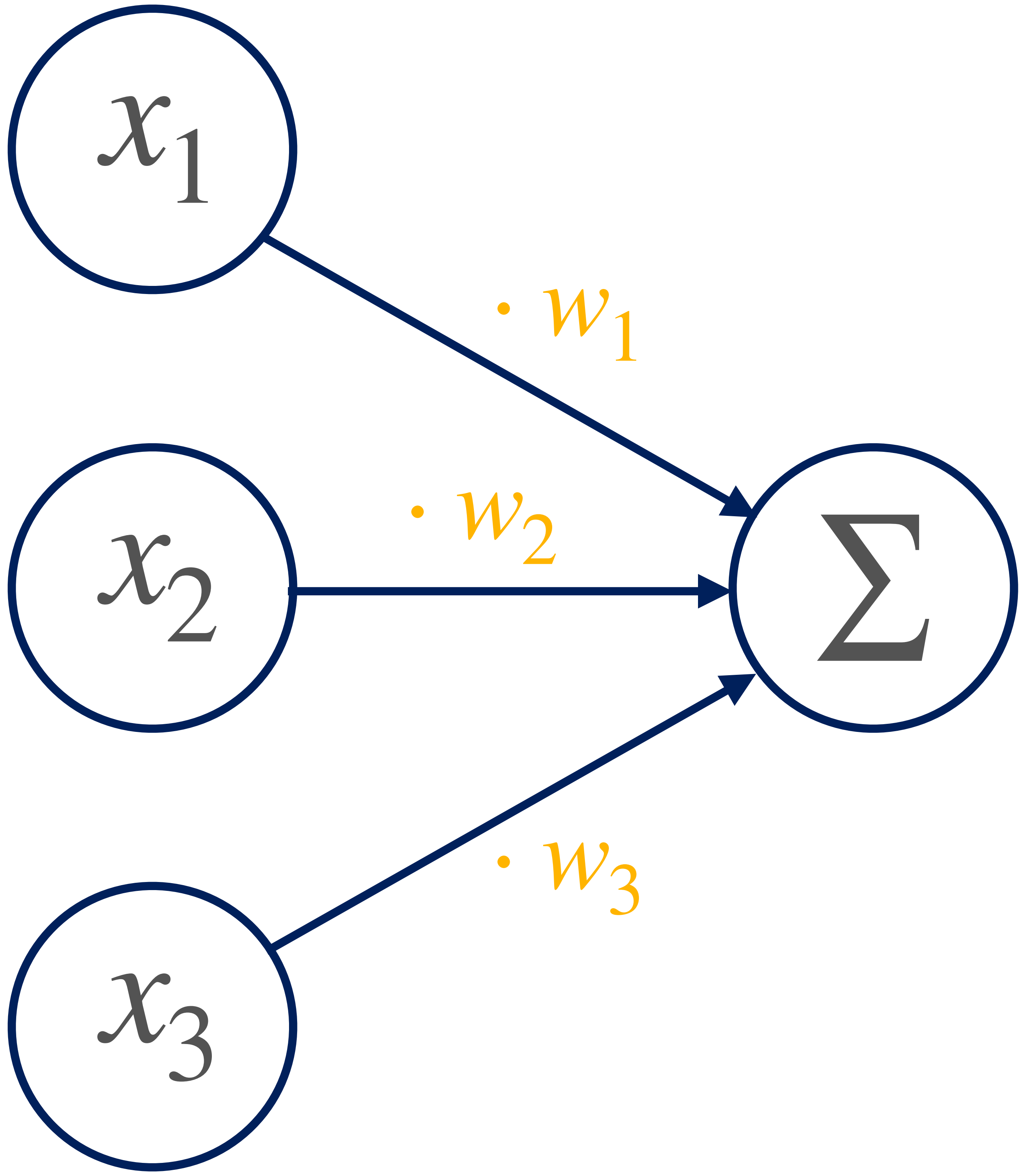

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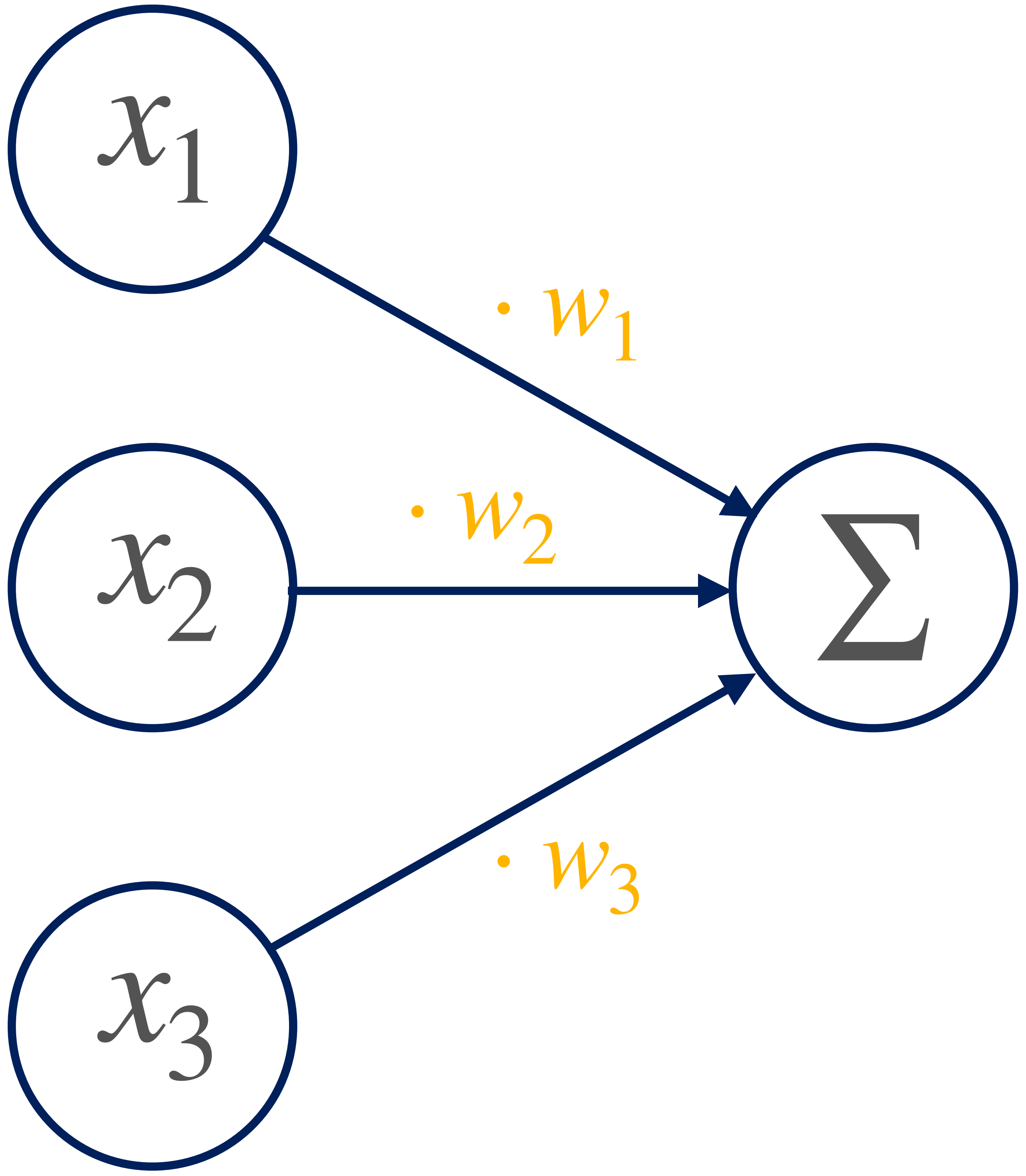

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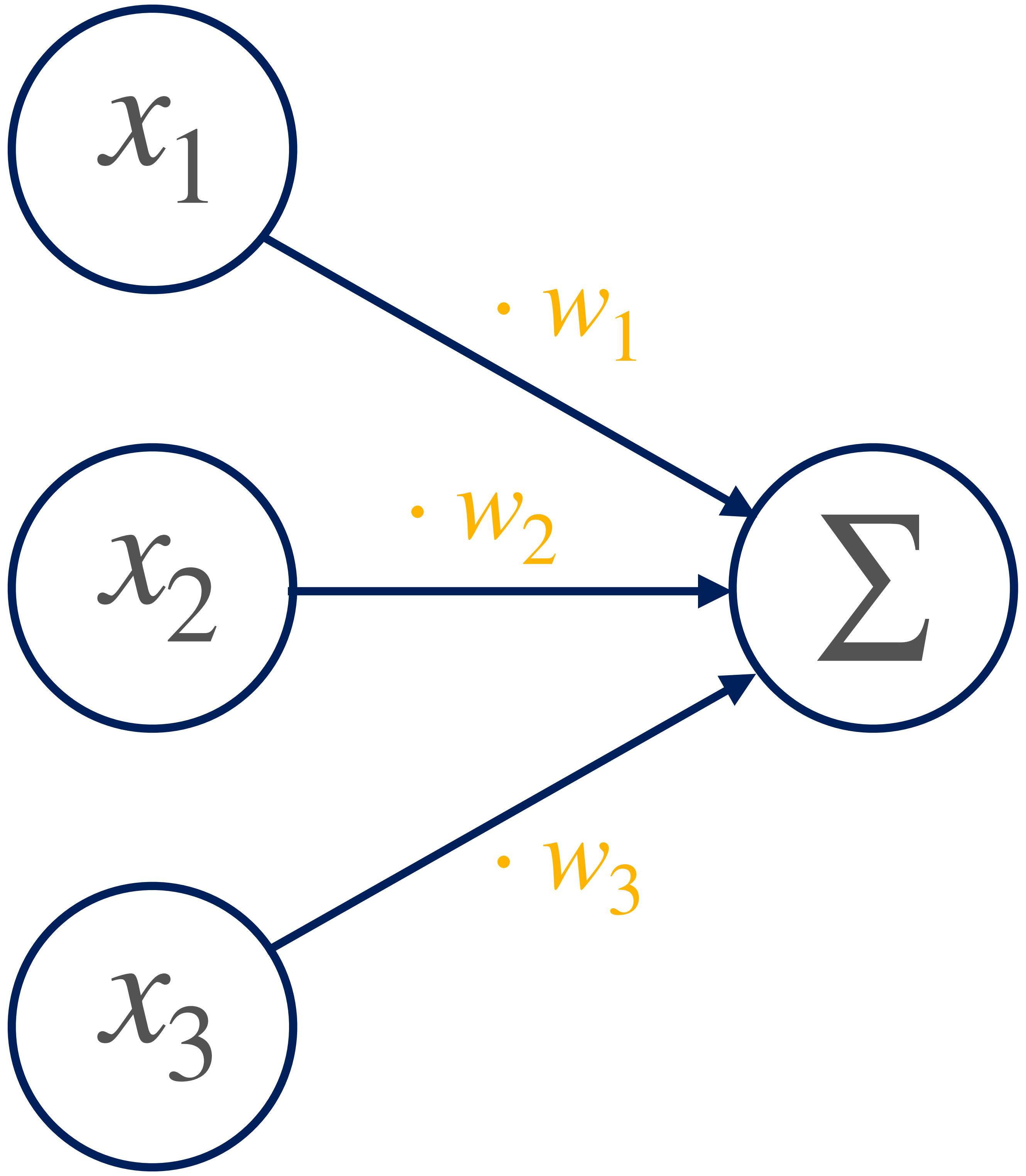
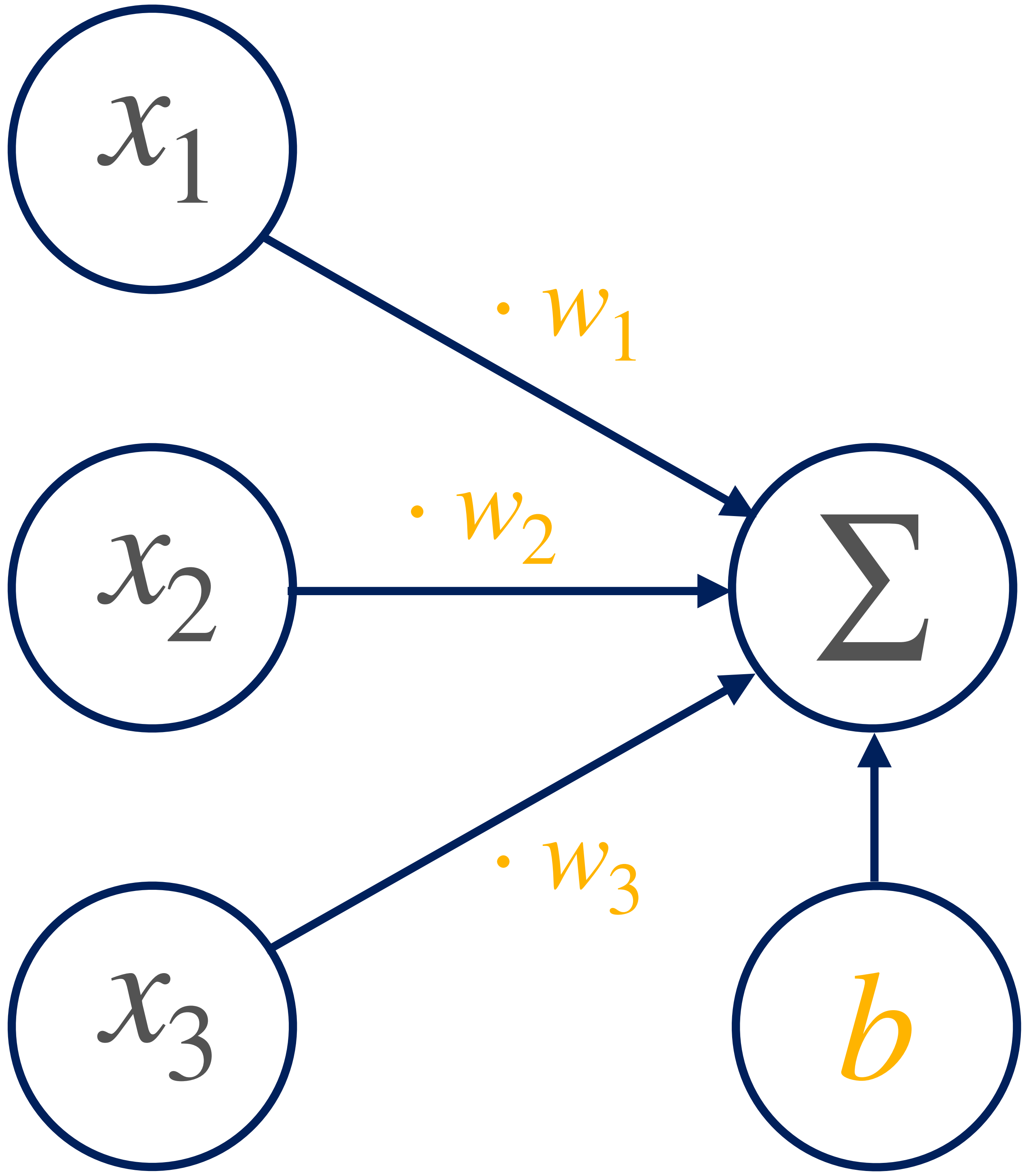
  - We'll introduce this learning next time



A Small Correction



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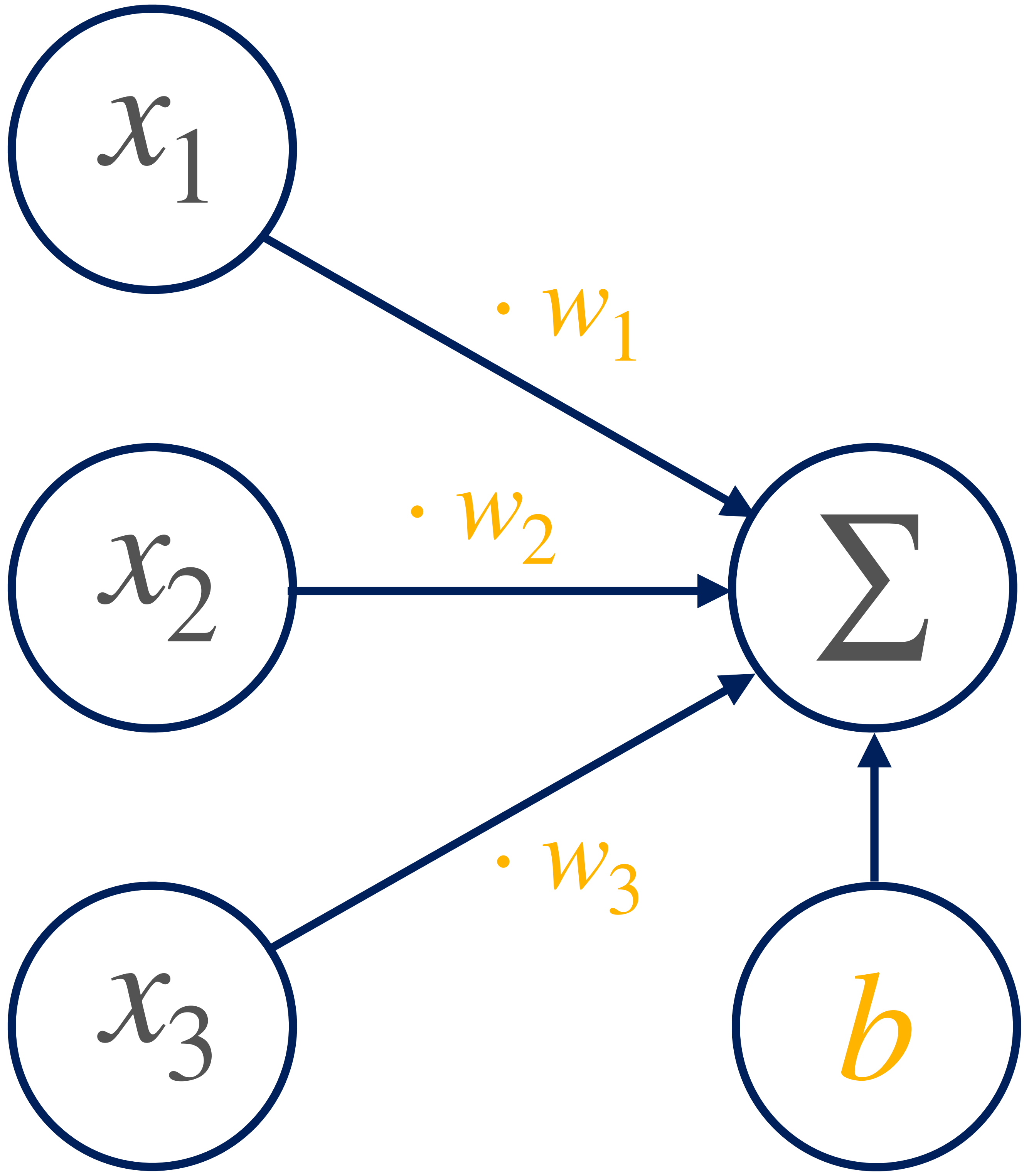

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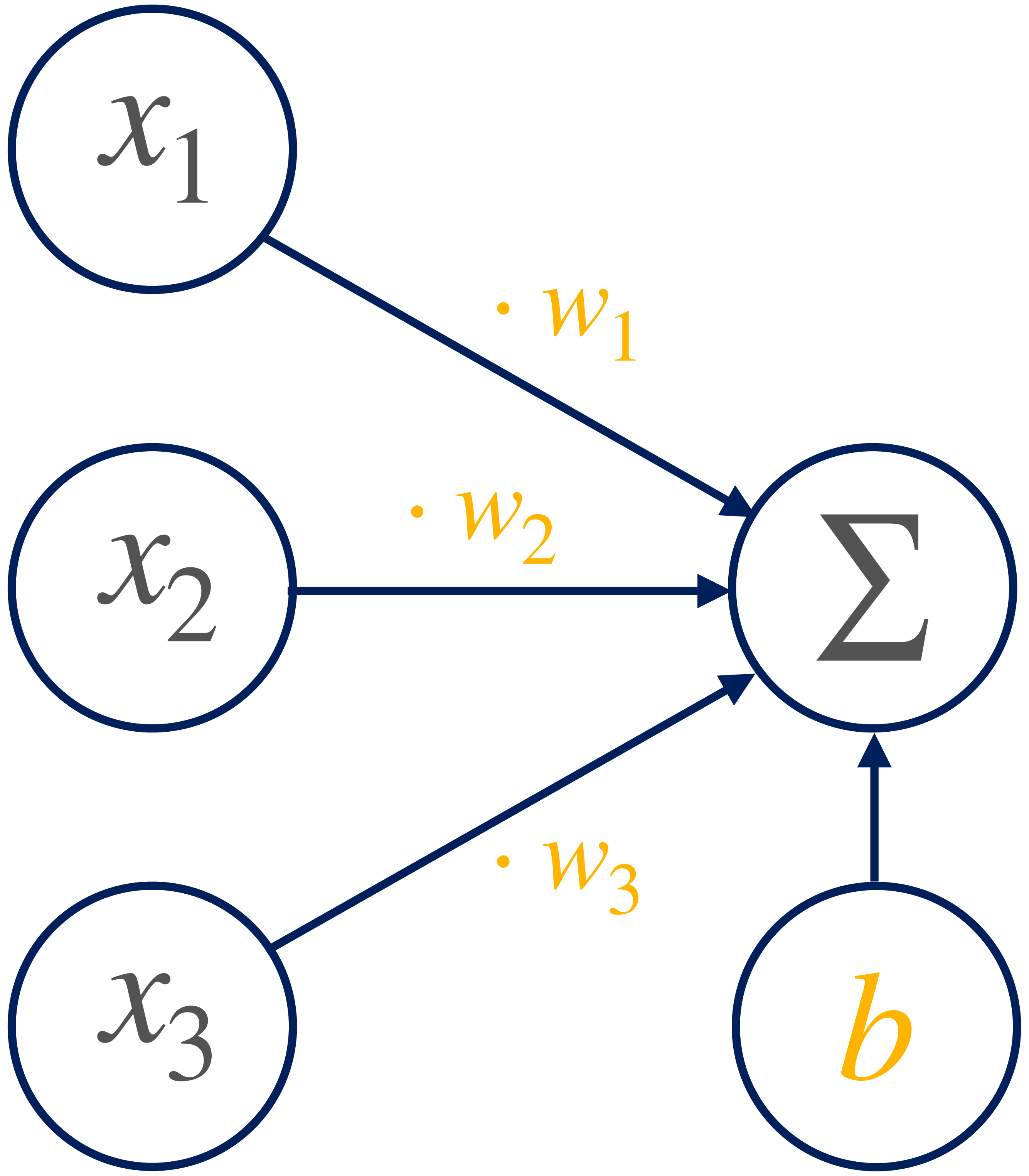

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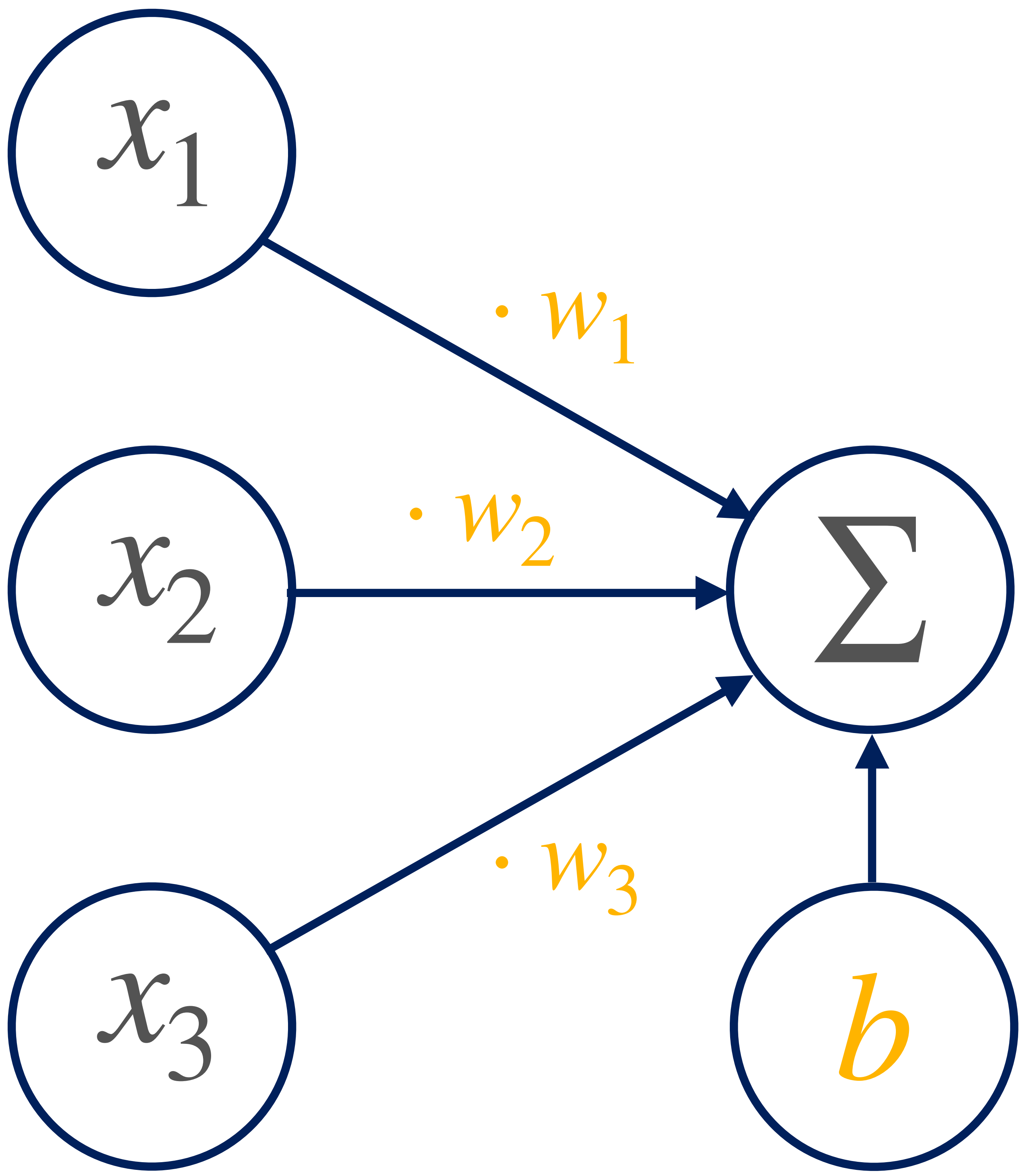
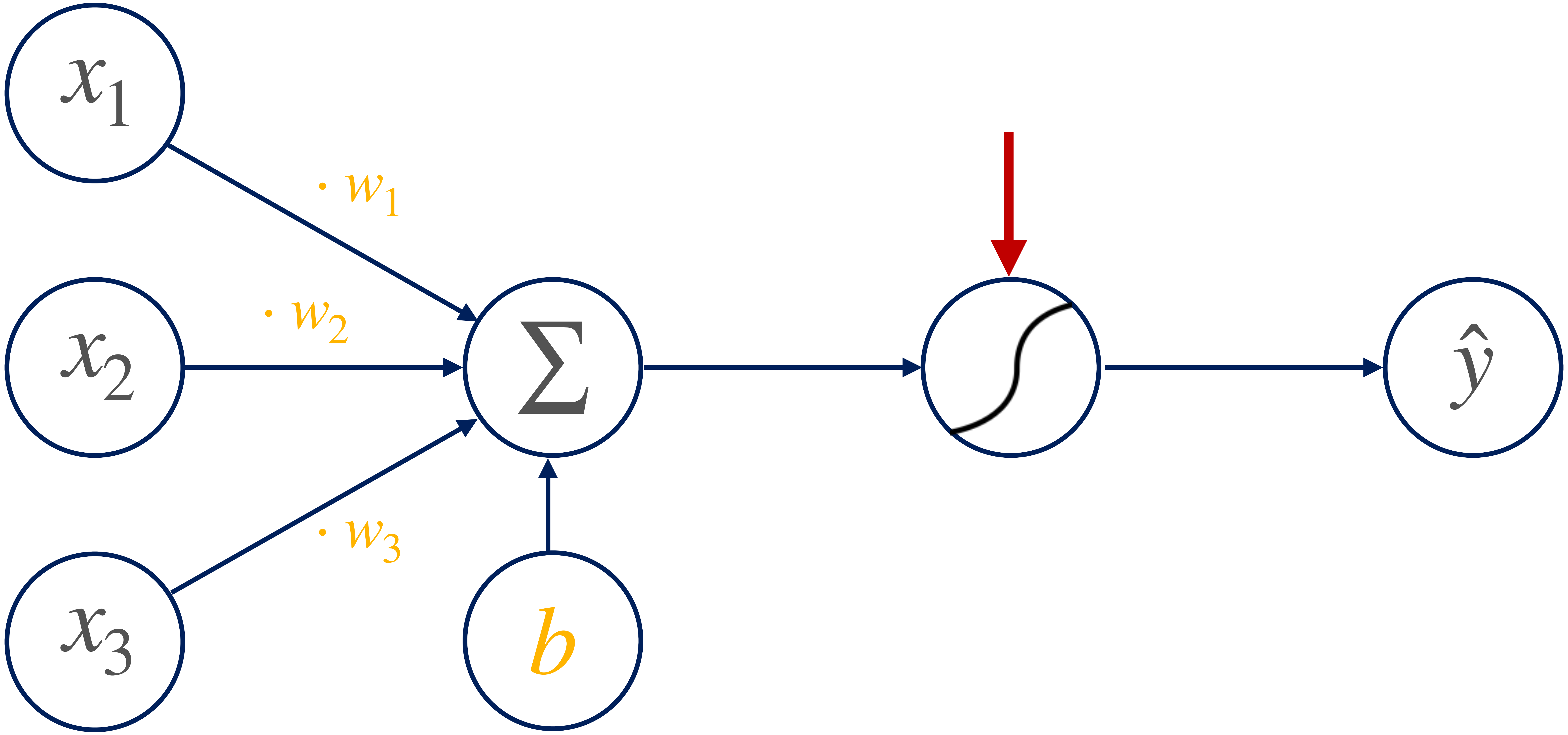
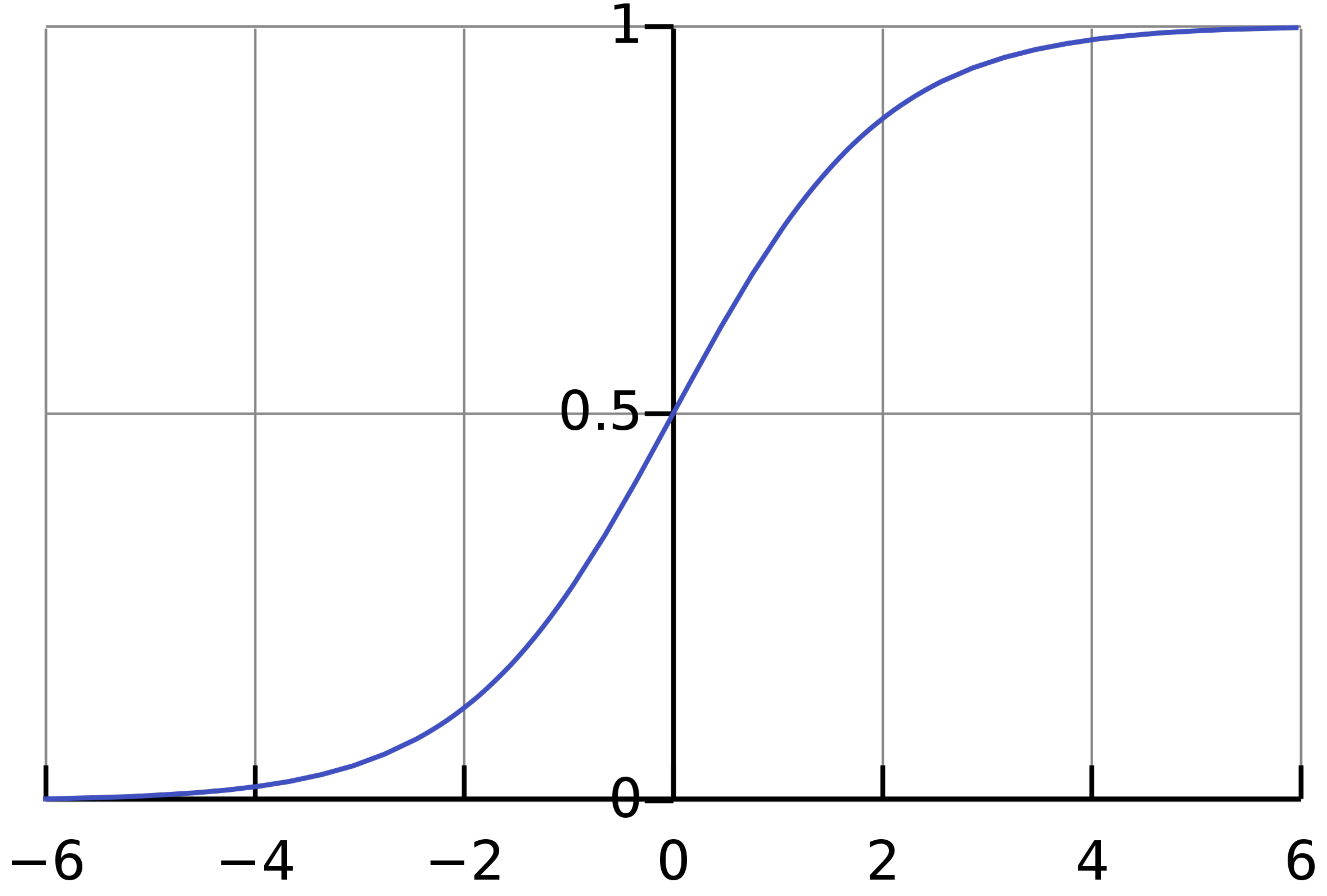


Introducing the Sigmoid



Sigmoid Function

$$\sigma(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{e^x+1}$$



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  - (Sigma σ is the Greek letter S)
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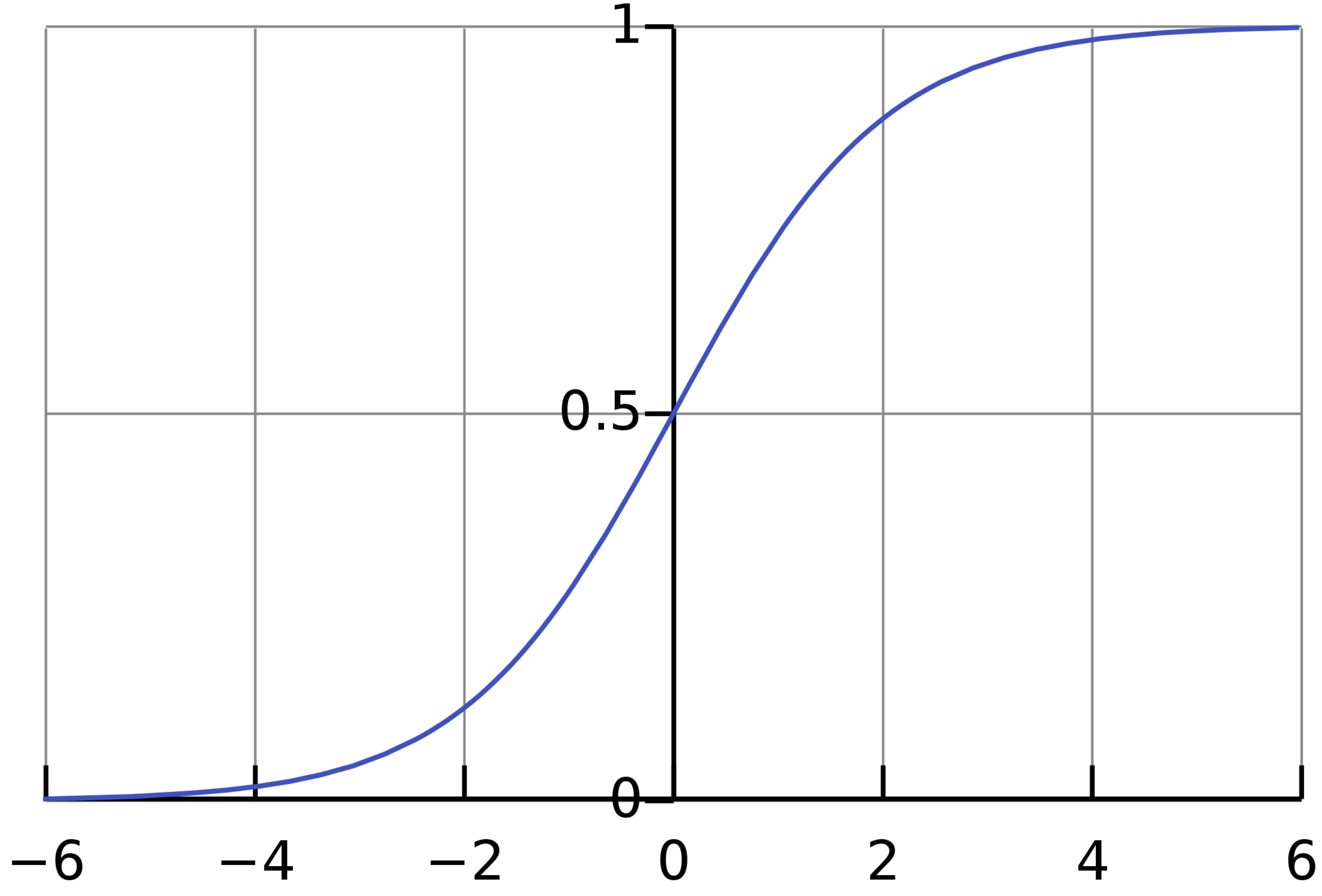

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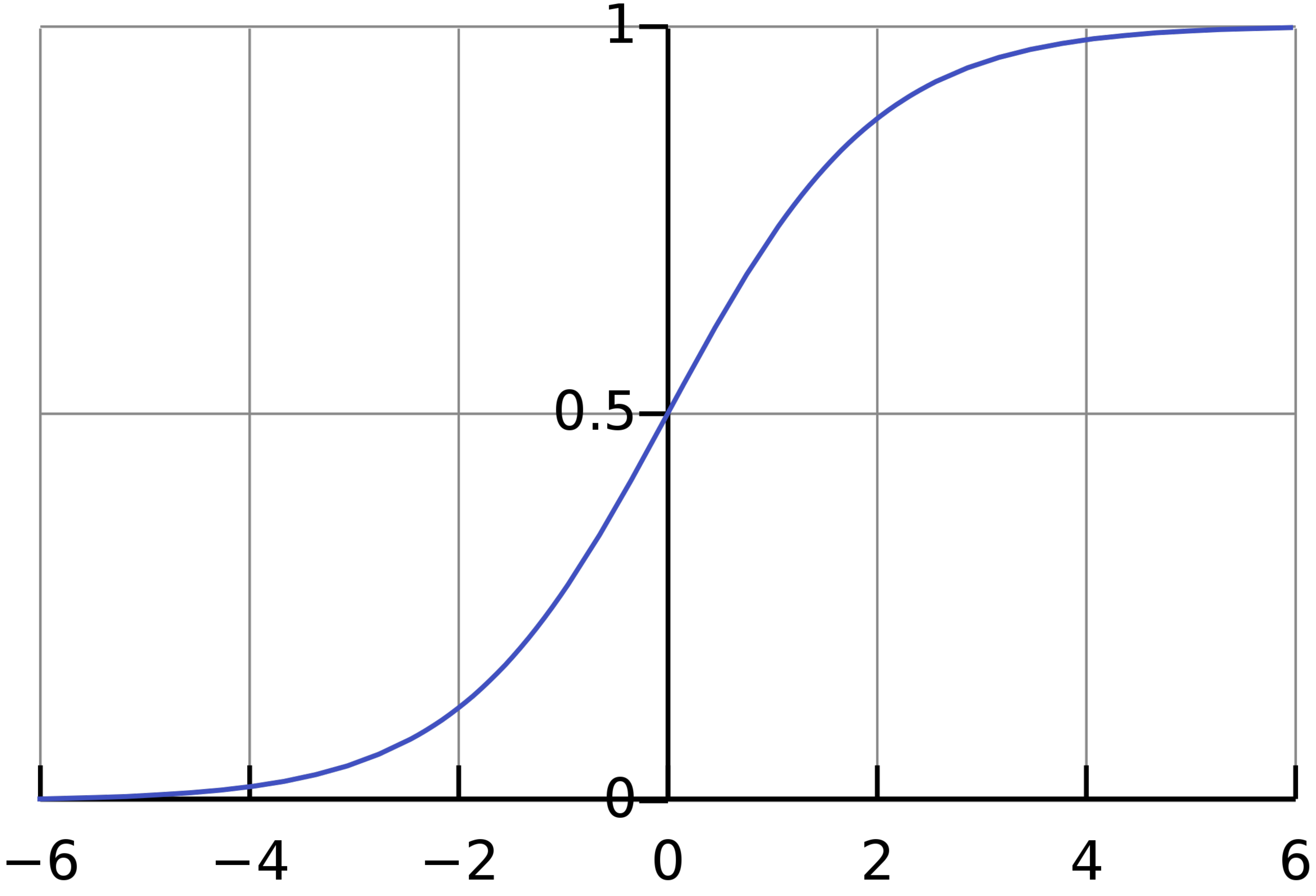

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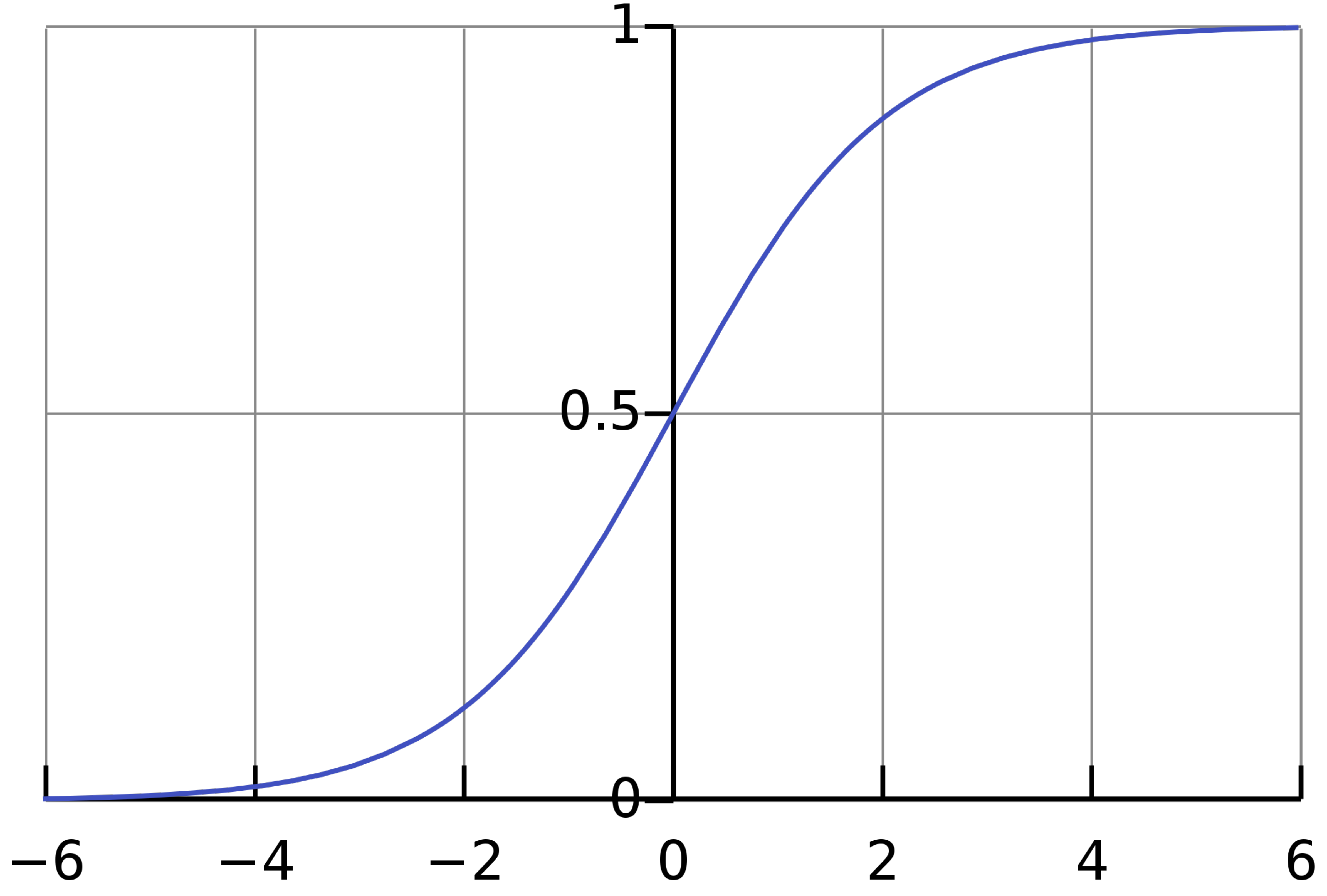
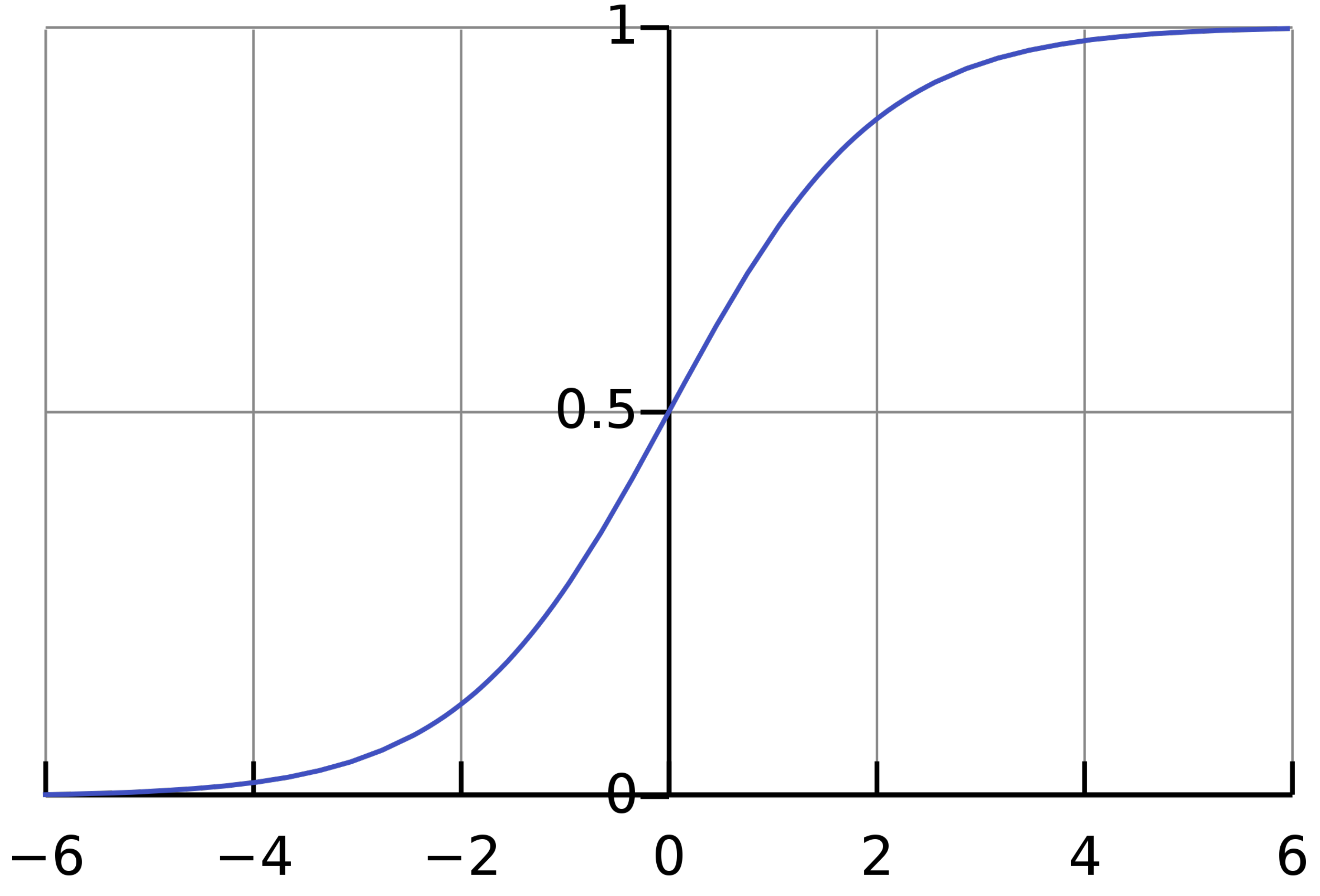
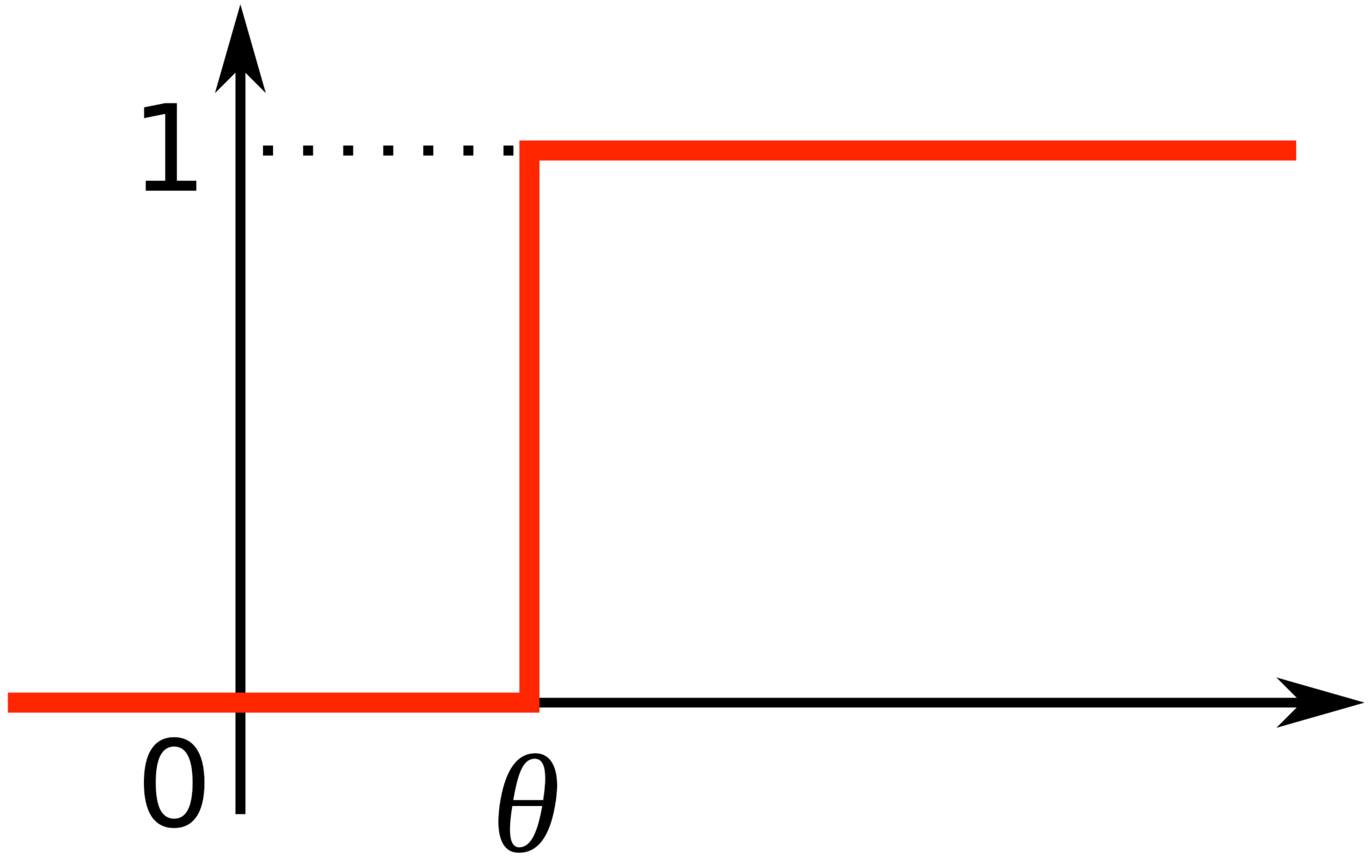
- Works as a **soft threshold**
  - $\sigma(x)$ is **close to 1** when $x > 0$,
  - **close to 0** when $x < 0$
  - Output range: (0, 1)



Sigmoid vs. Hard Threshold

- Key difference: Sigmoid is **smooth**, i.e. has a **well-defined derivative**
- (This will be important when we get to **learning algorithms**)





Binary Classification

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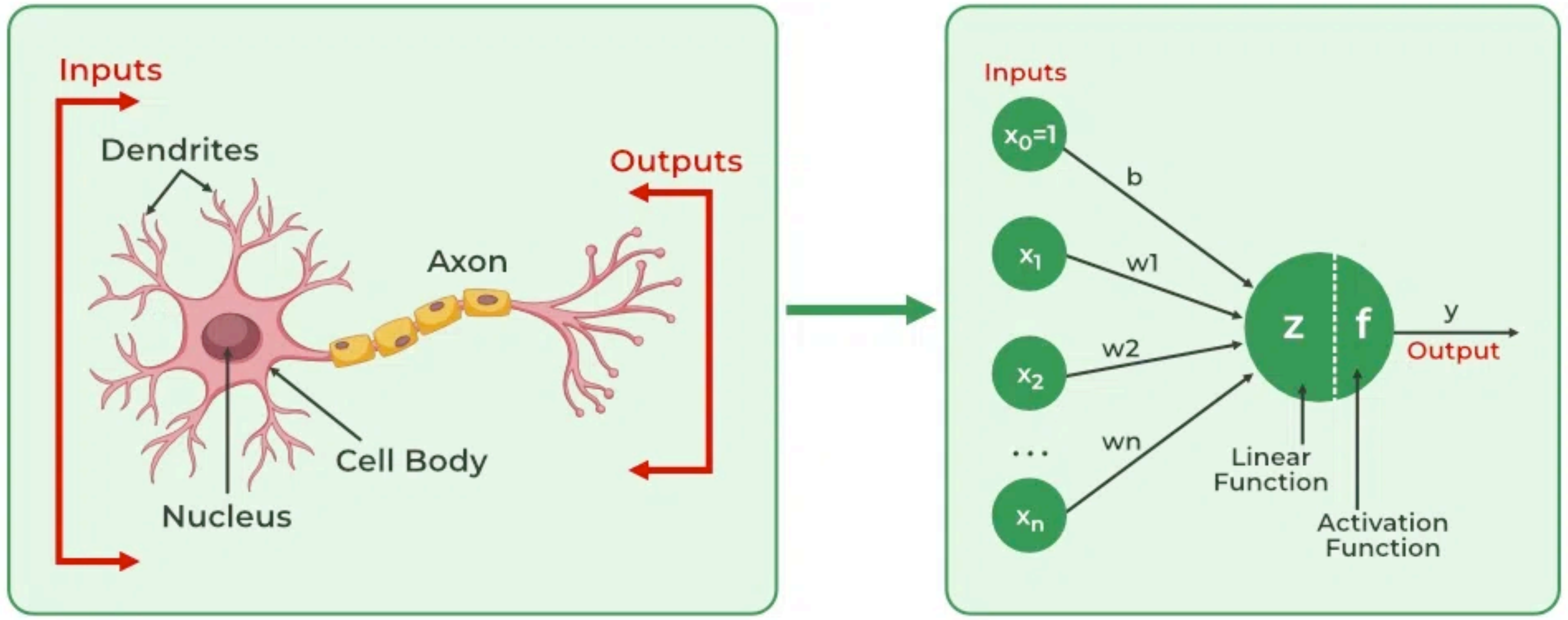
- Speeding ticket example: suppose $\mathit{Perceptron}(38) = \hat{y} = 0.92$
  - This means the Perceptron gives a **92% chance of getting a ticket**

"Artificial Neuron"



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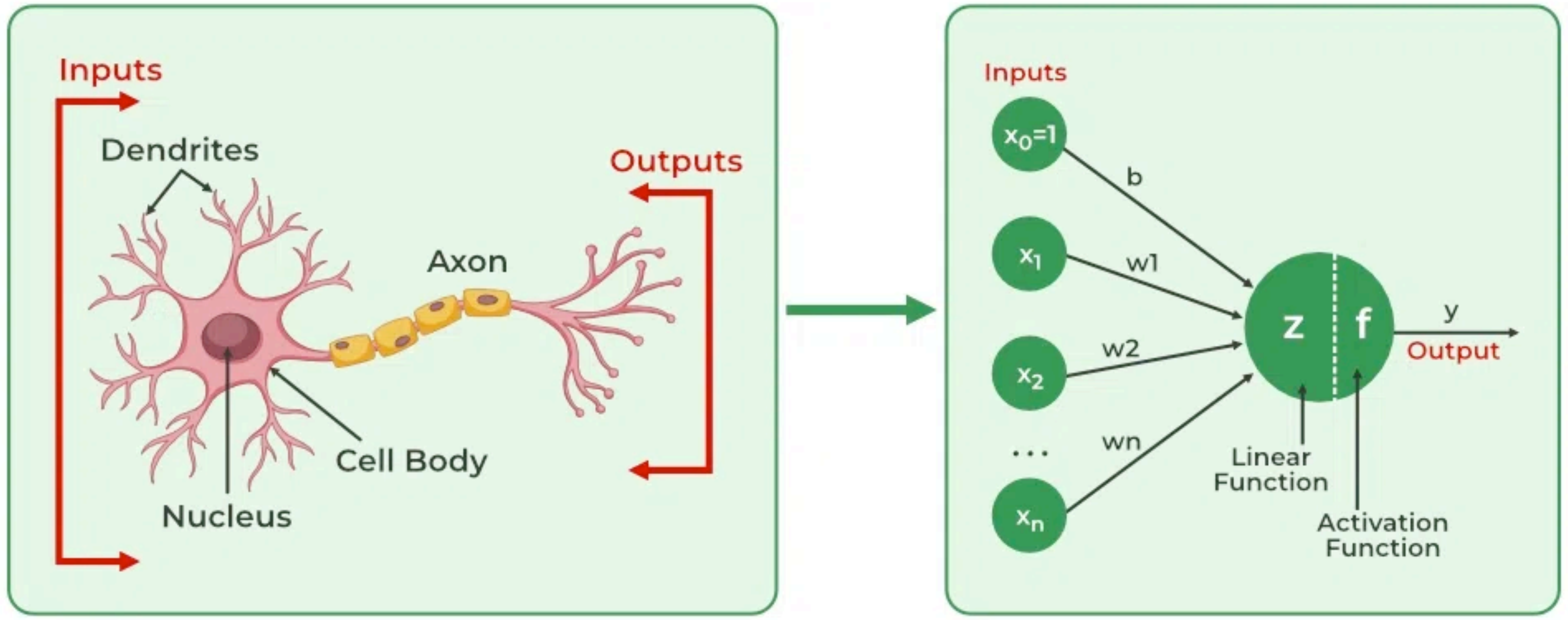

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- A neuron...
  - Takes in **input signals** from other neurons
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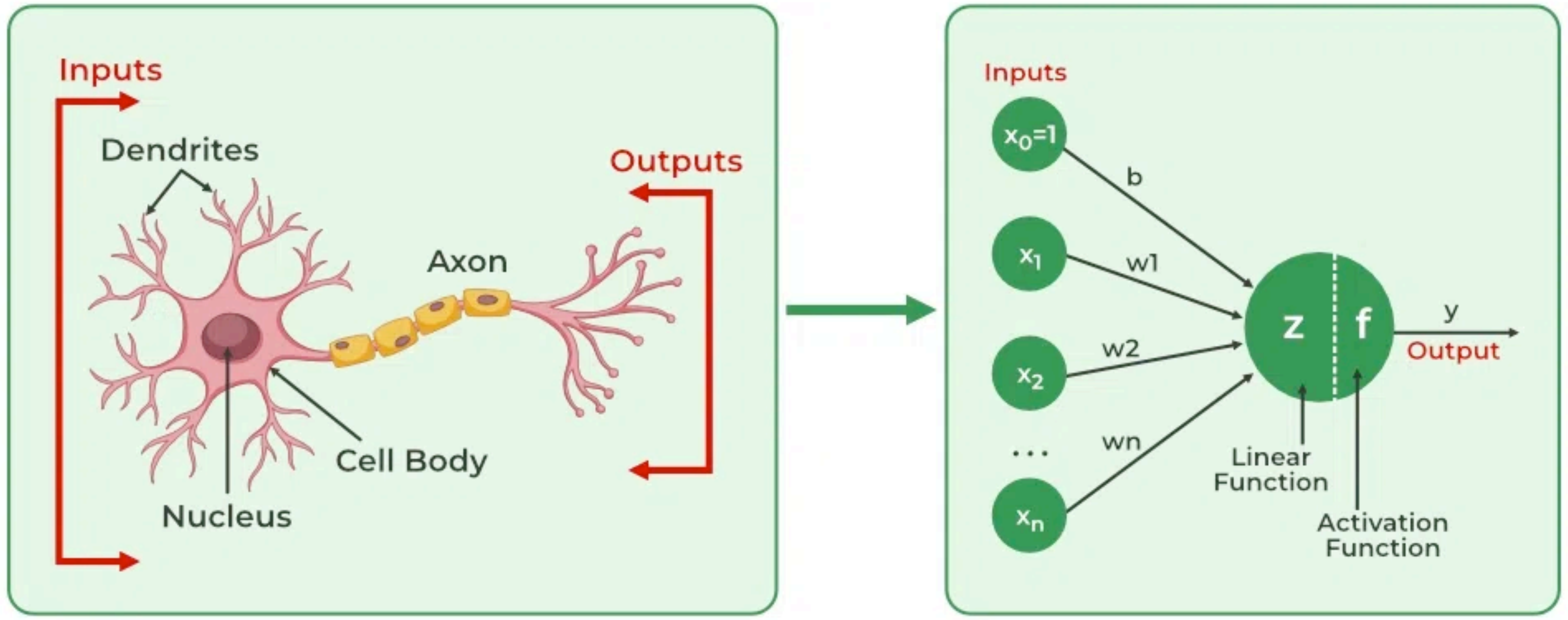

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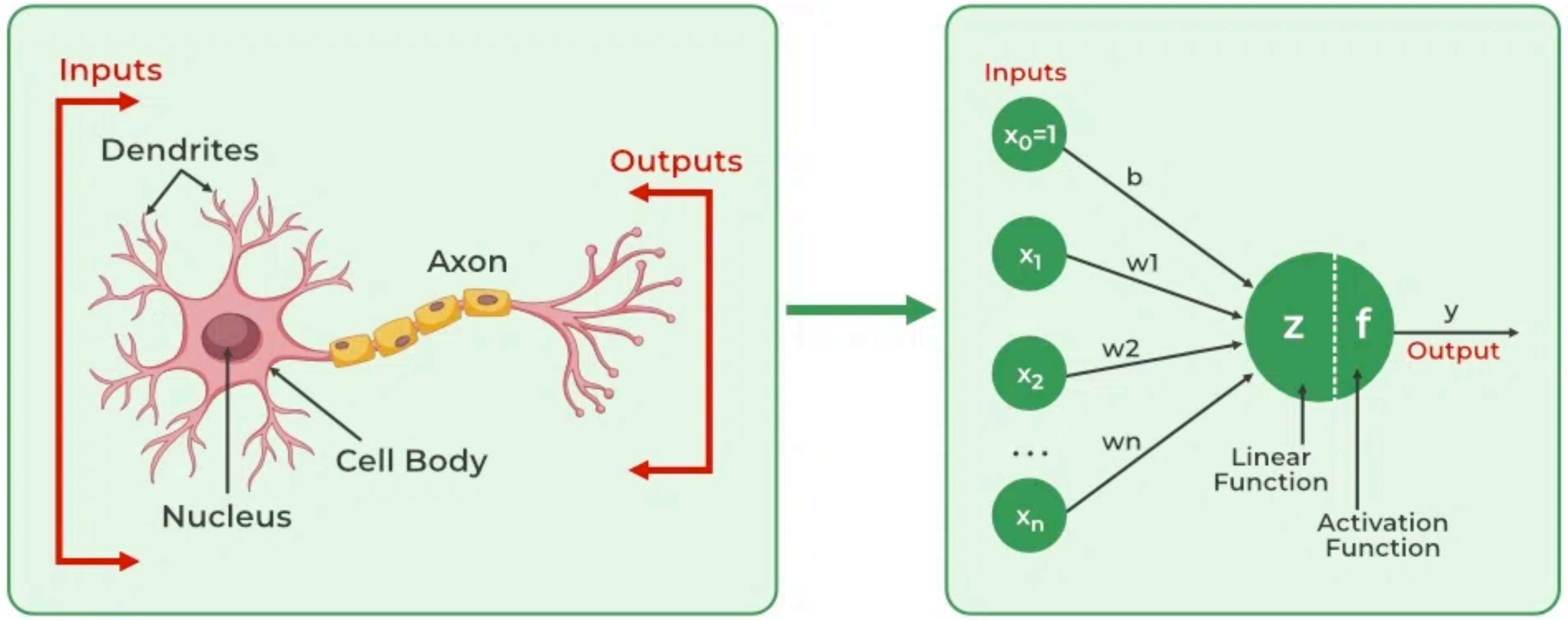
- **Networks of neurons** can model sophisticated functions



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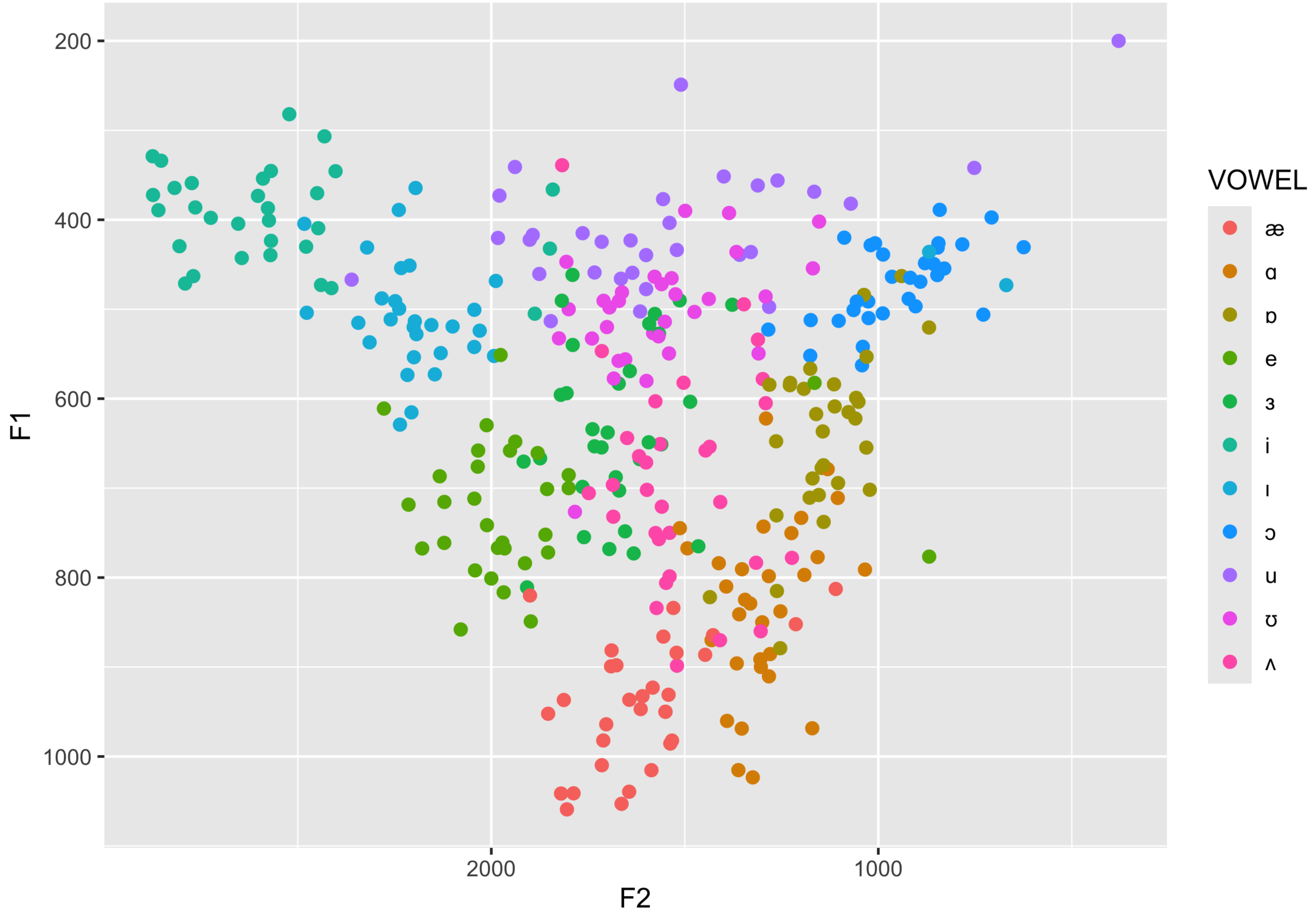
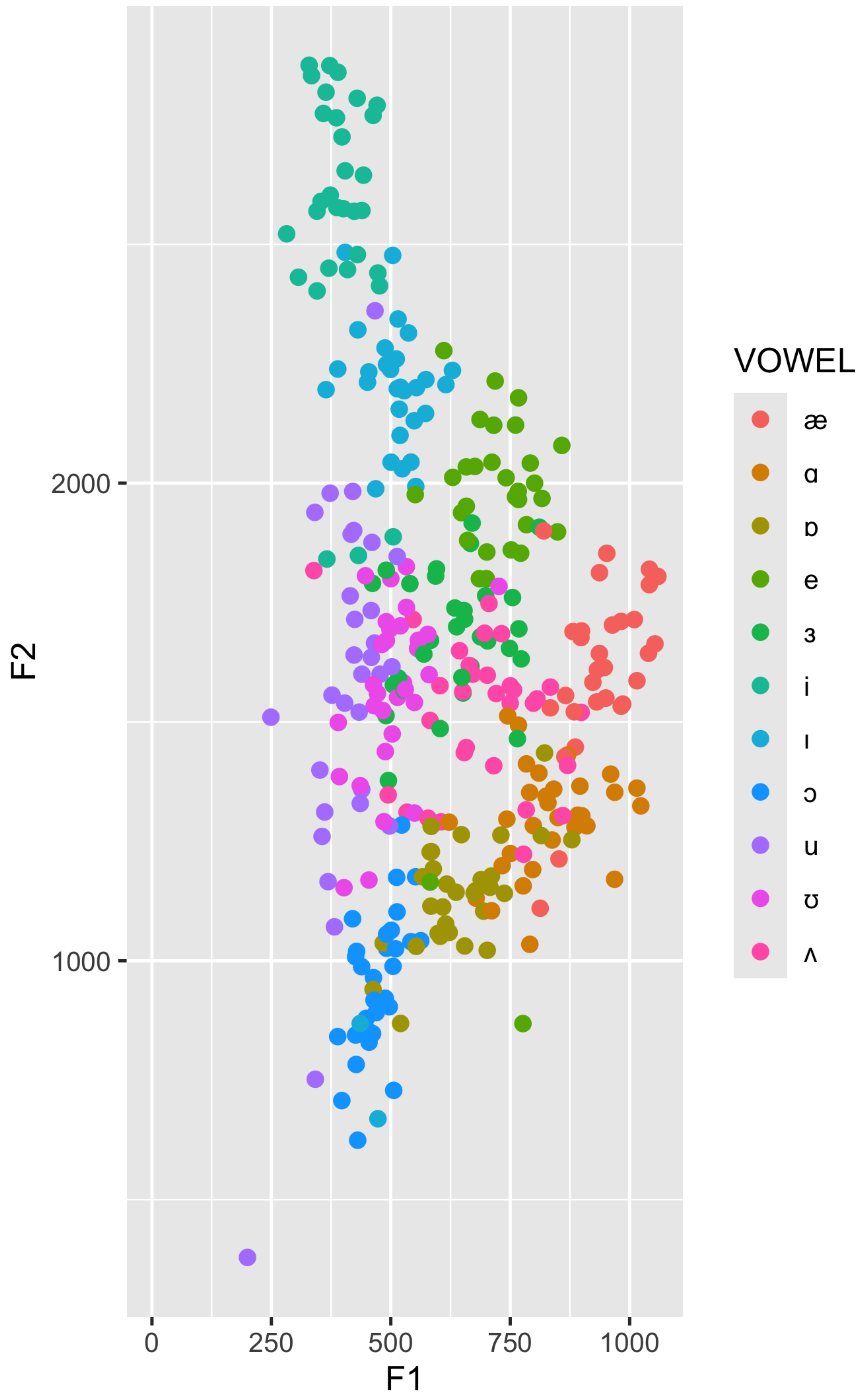
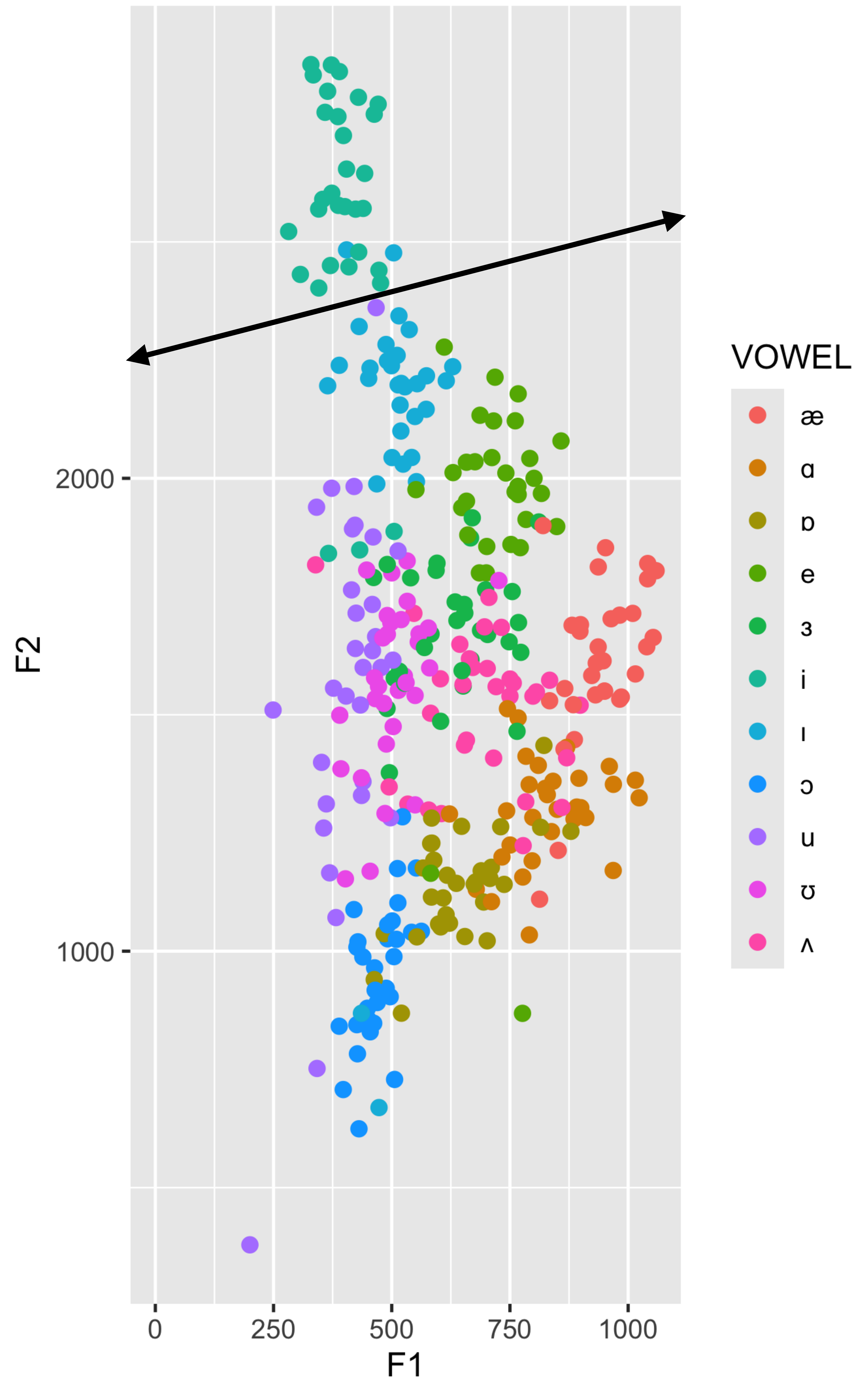
Perceptron and Linear Separation

- Example: **classifying a vowel** by its formant frequencies
- Vowels can be characterized by two component frequencies called **formants** (F1, F2)
  - These (very roughly) correspond to notions of "height" and "backness"
- Can we **identify the vowel [i]** with binary classification?



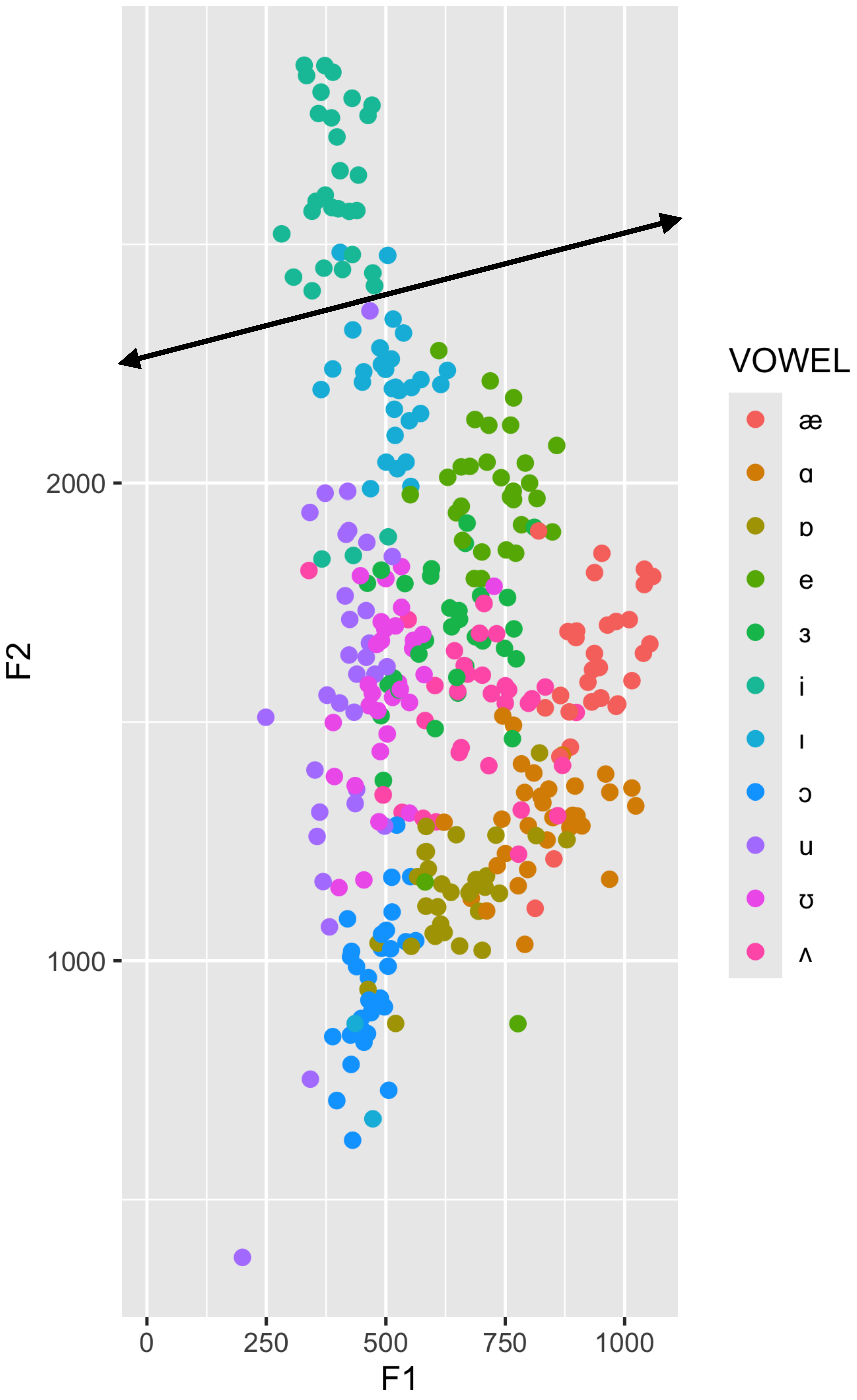
- Challenge: draw a **straight line** that **separates** the vowel [i] from **all other vowels**
  - Can a perfect boundary be drawn?
  - Draw an **imperfect one** if not
- Can we get a **formula** for this line?





F2
2000
1000
0
250
500
750
1000
F1
VOWEL
æ
ɑ
ɒ
e
ɜ
i
ɪ
ɔ
u
ʊ
ʌ

- We'd probably end up with something like this drawn line



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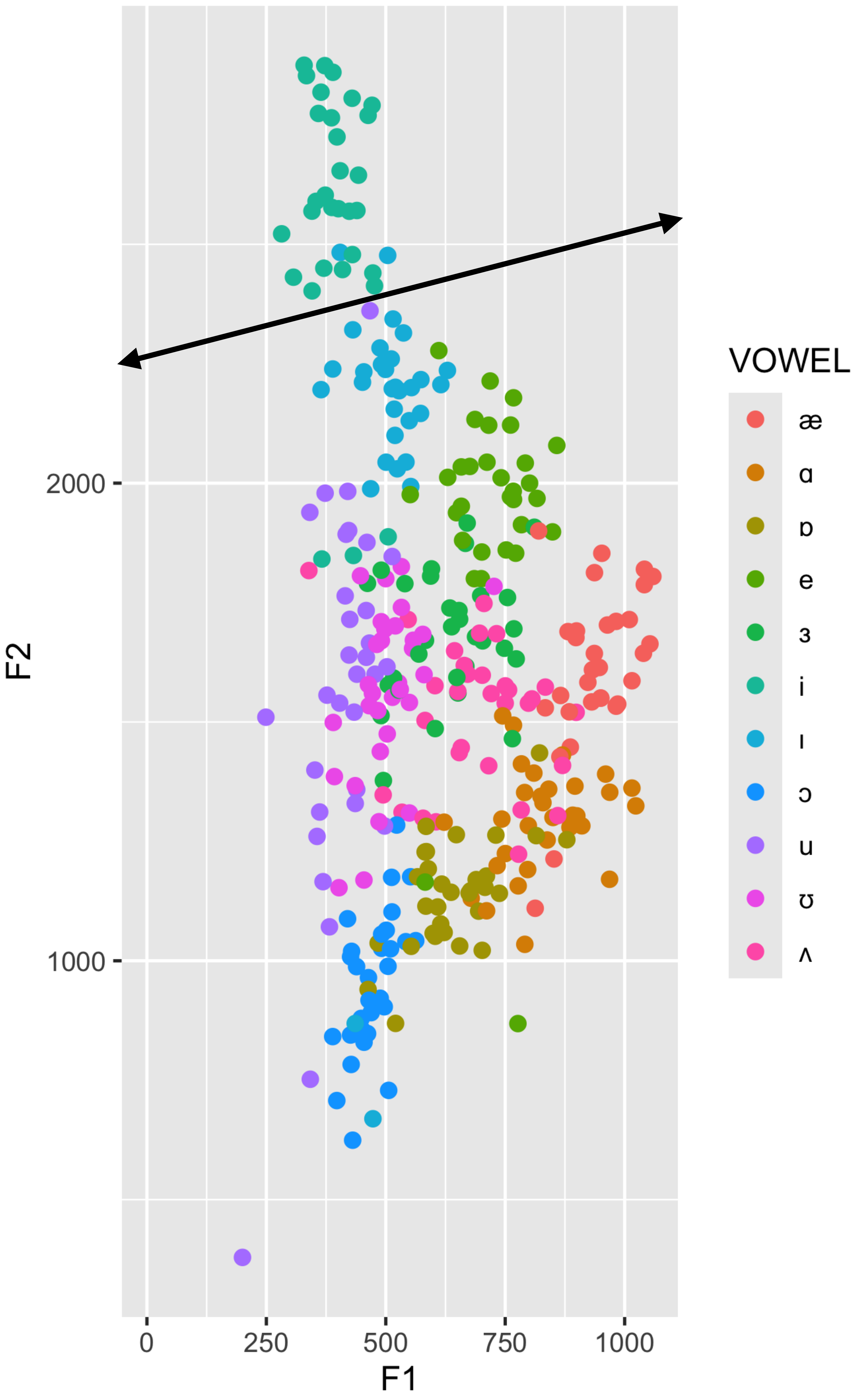
- Line formula: $y = mx + b$



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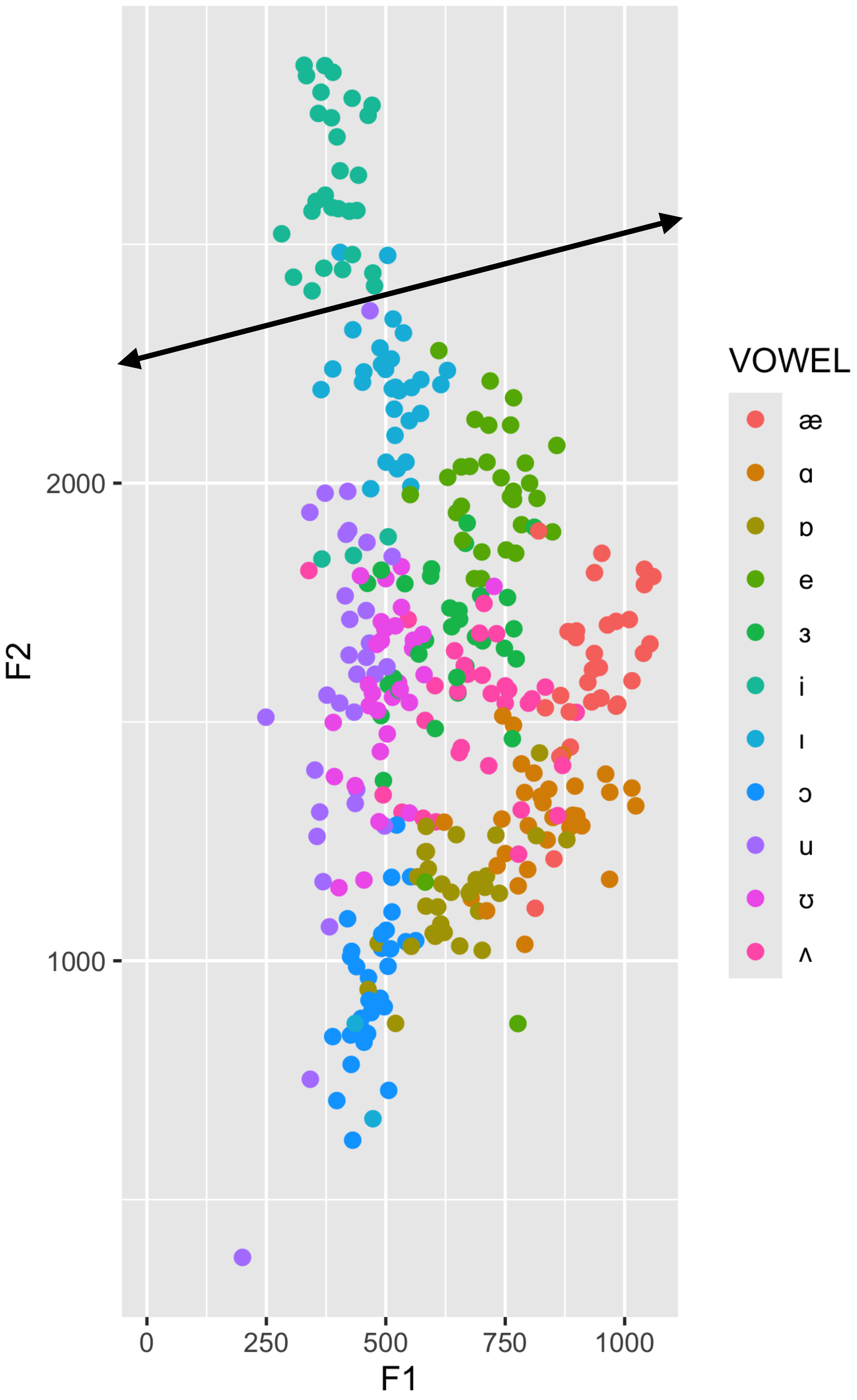

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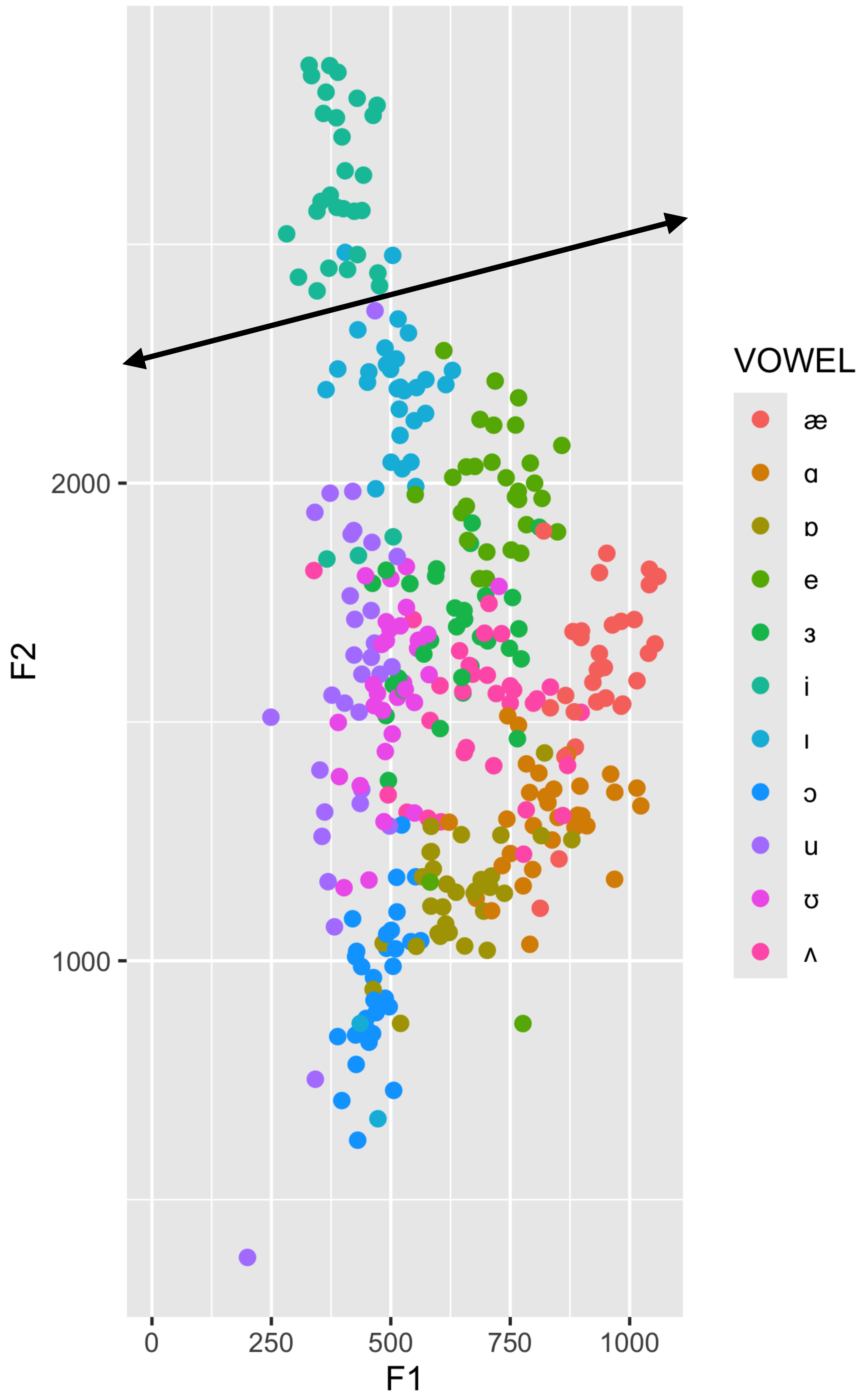

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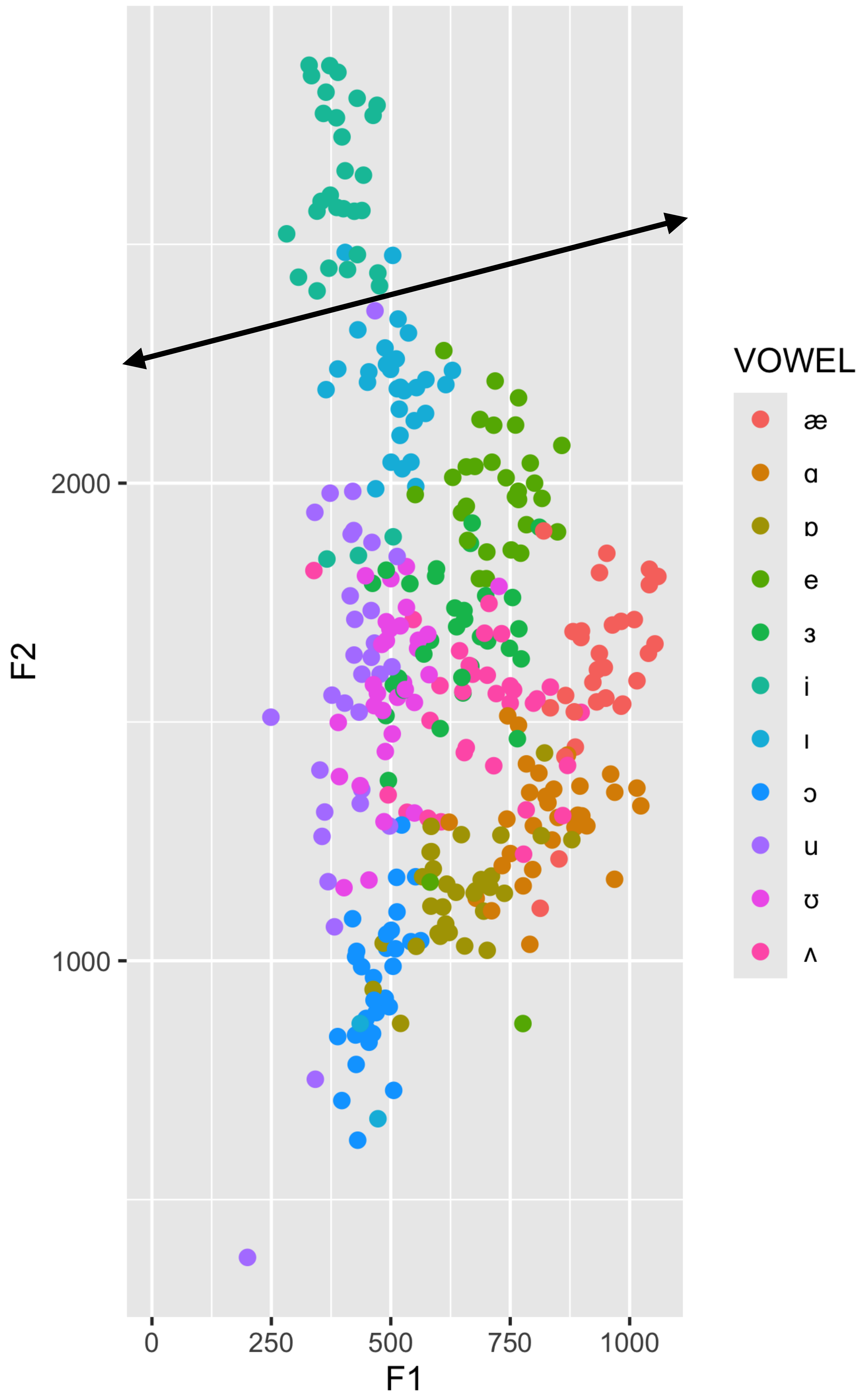

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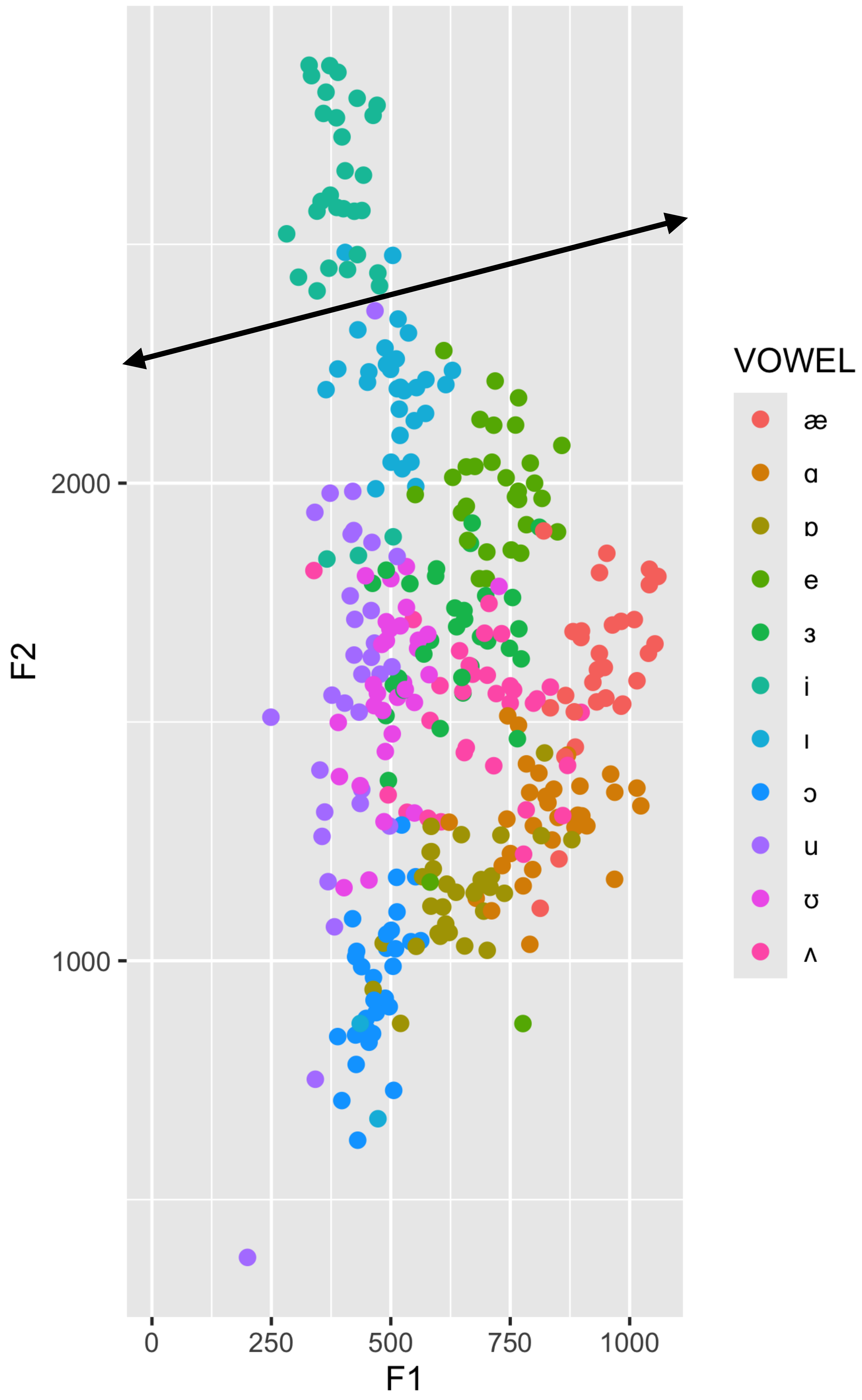

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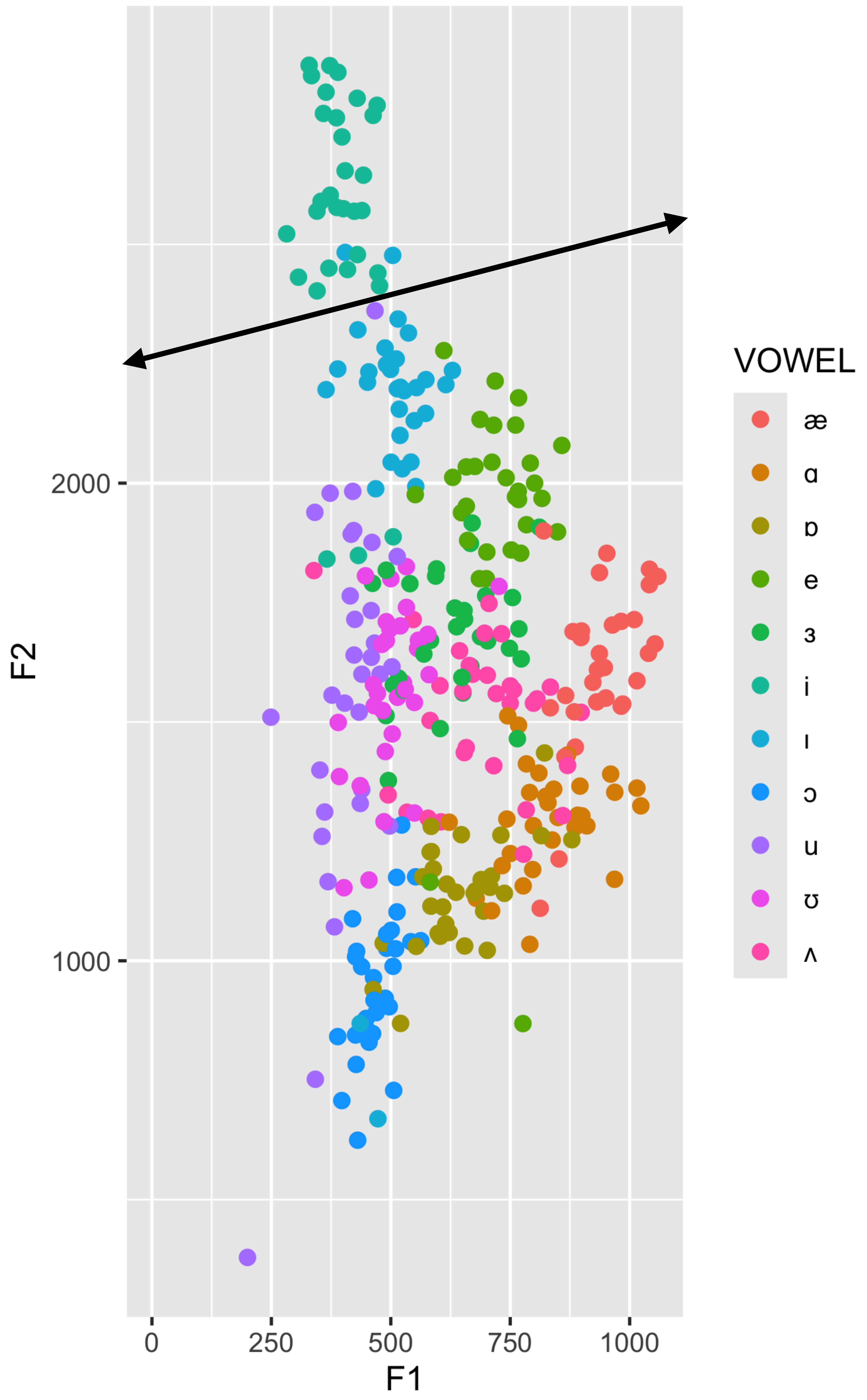

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$$1.0 \cdot \text{F2} + (-0.26) \cdot \text{F1} - 2265 = 0$$

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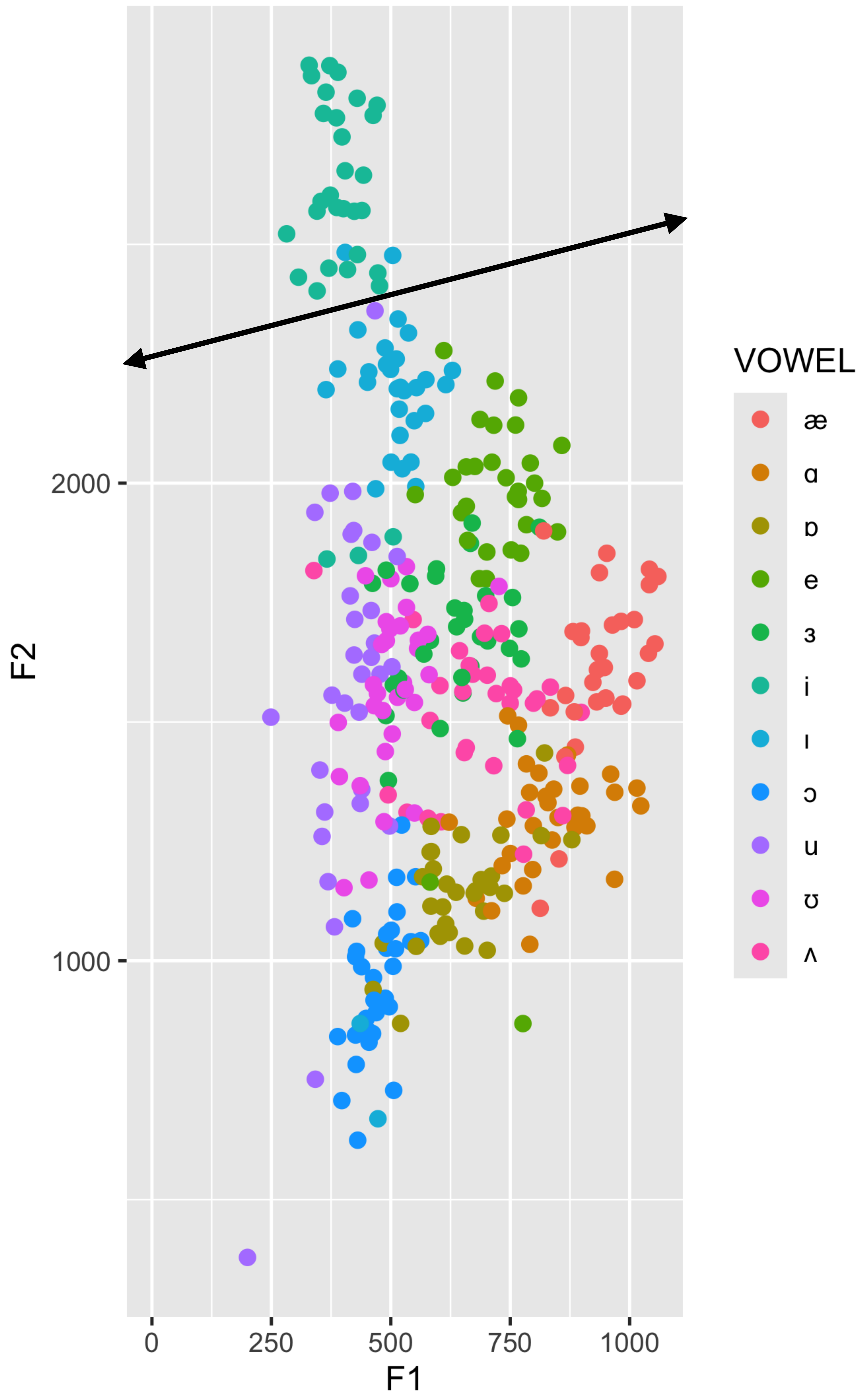

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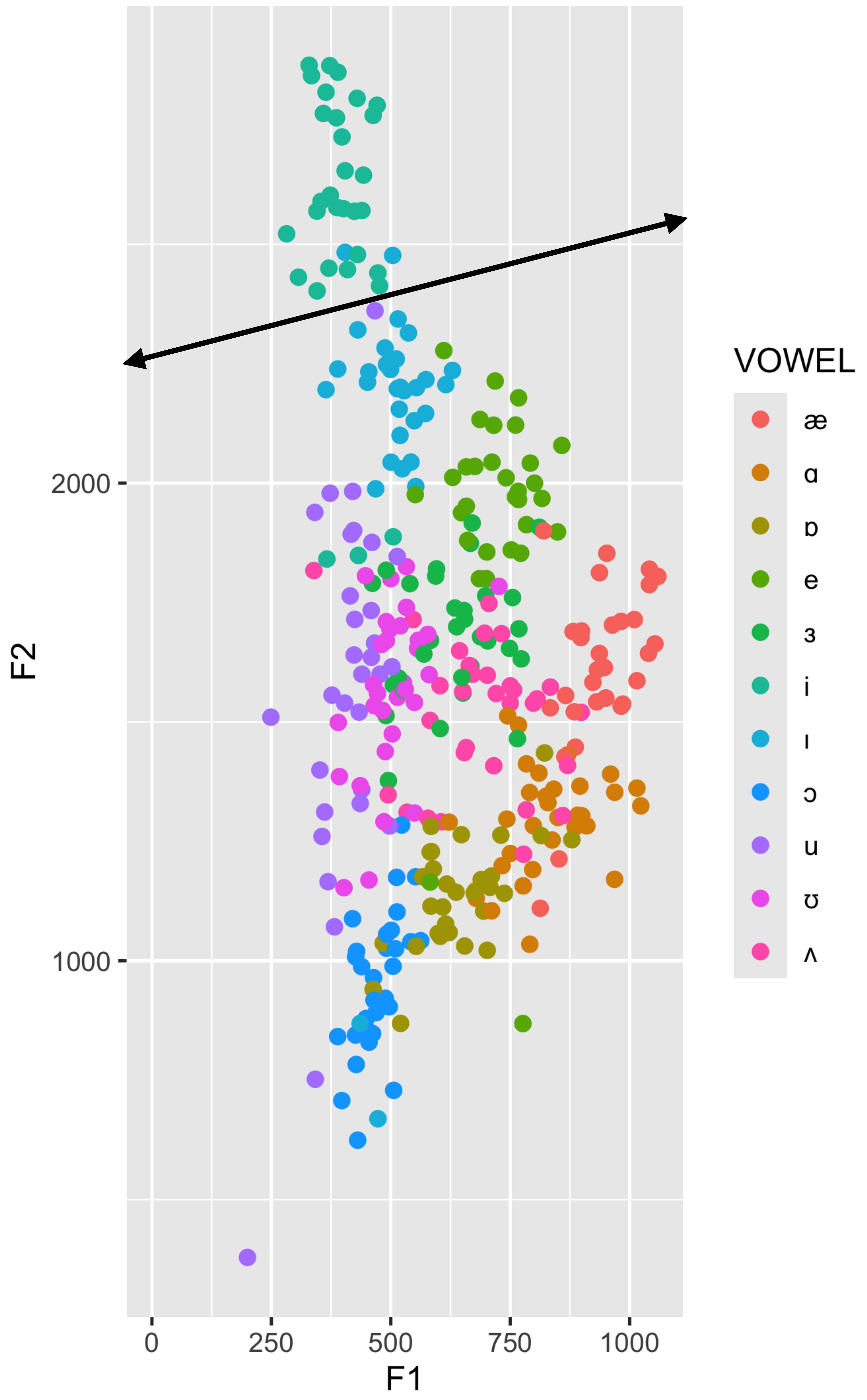
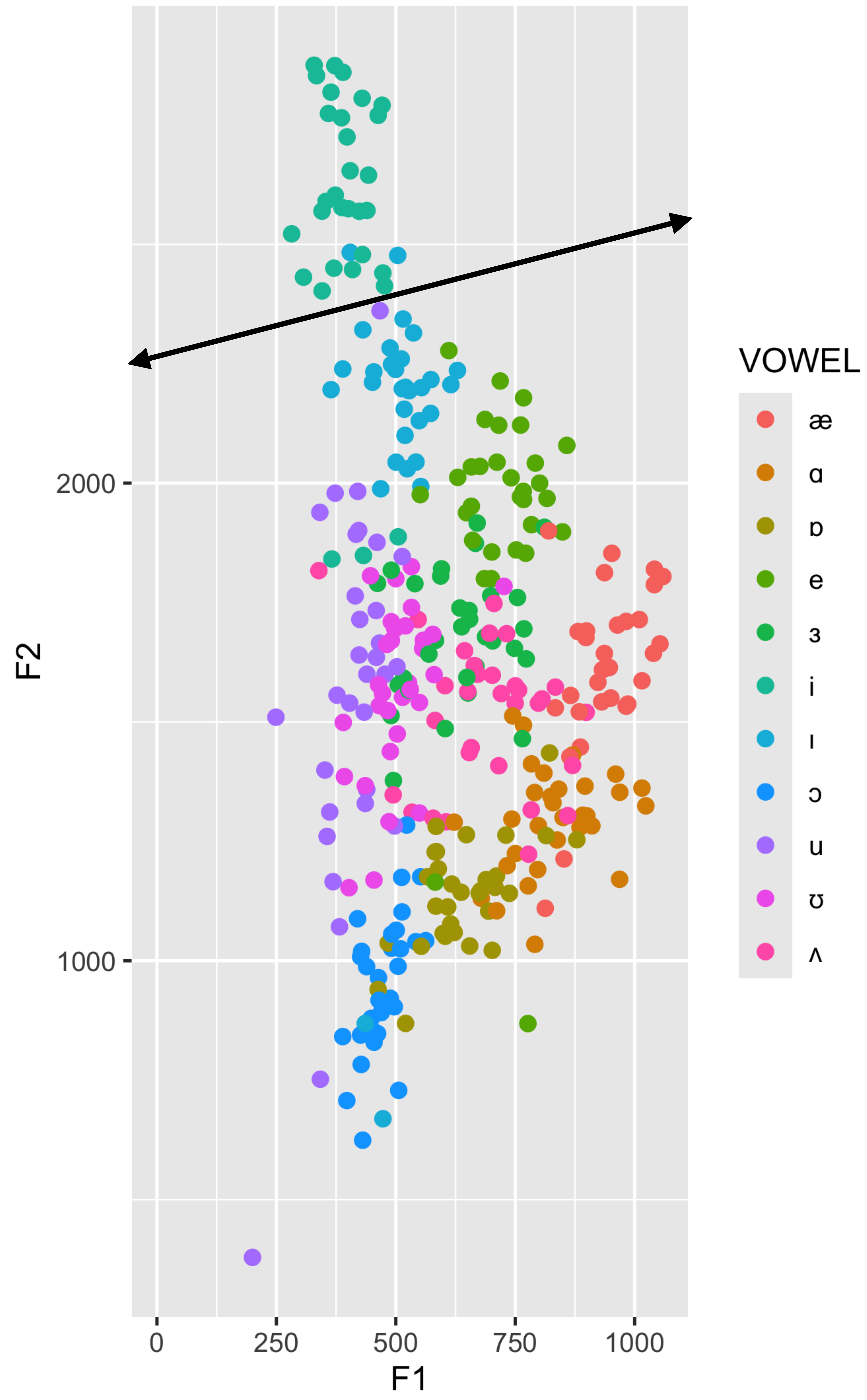
What does this remind us of?





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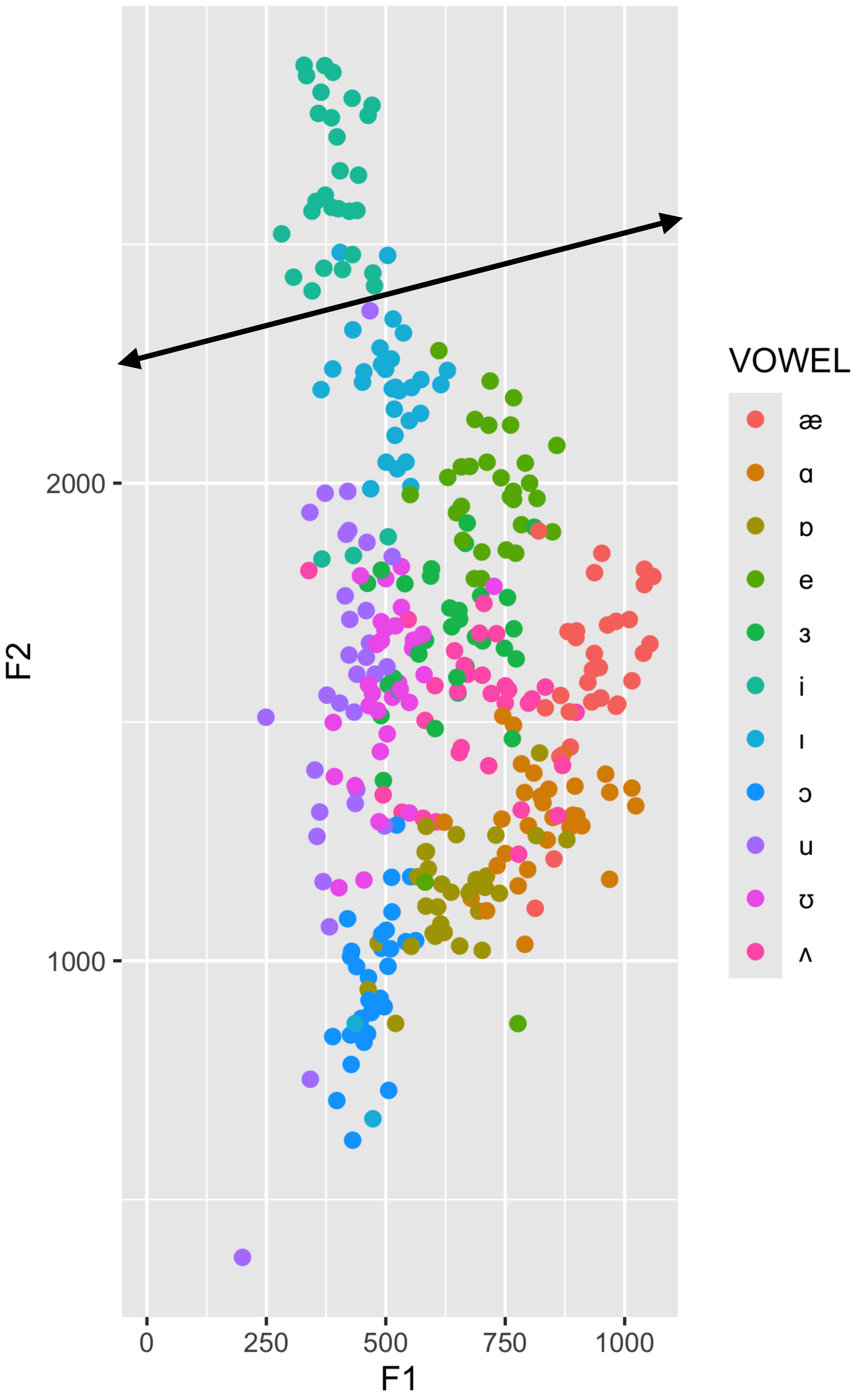

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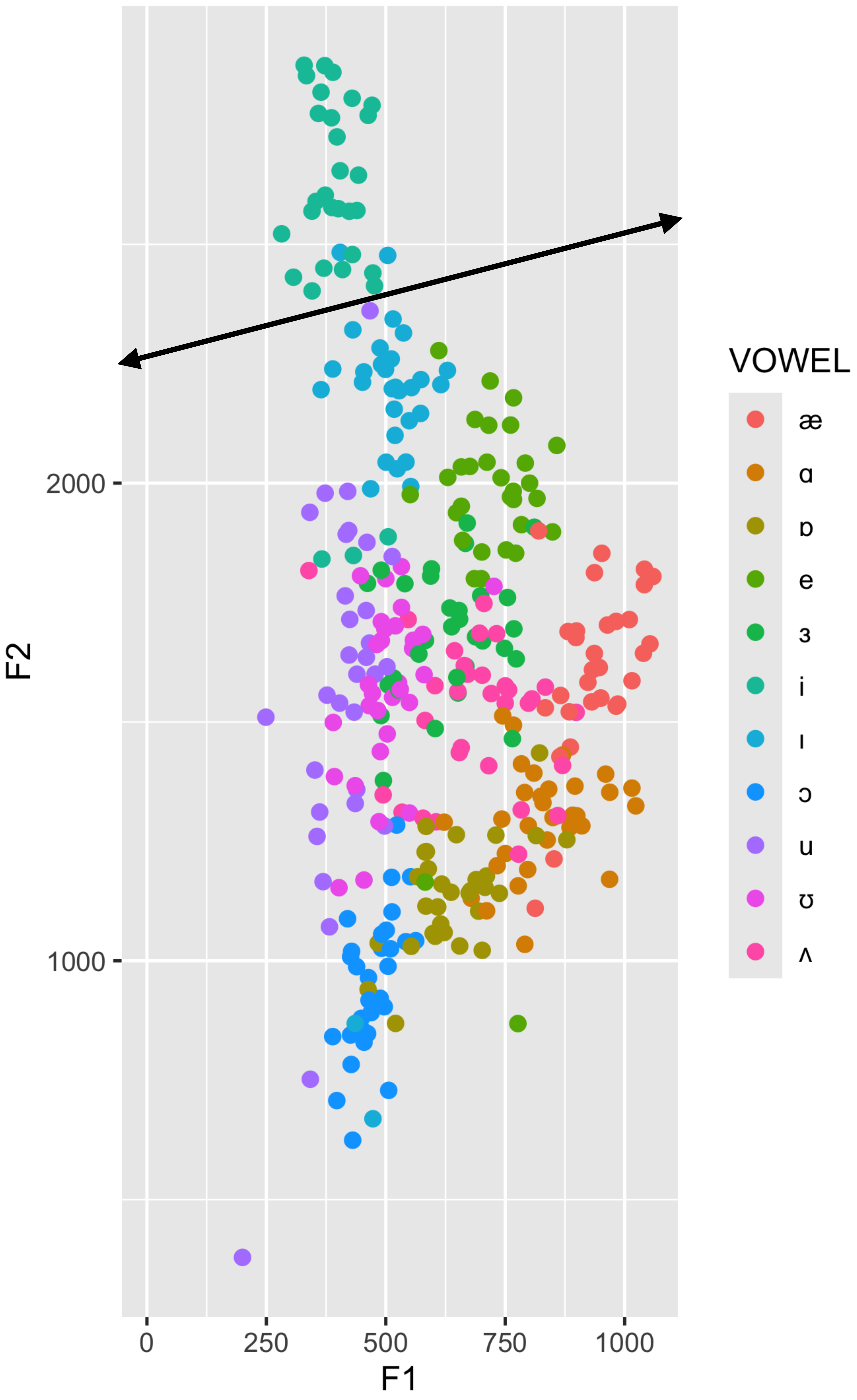

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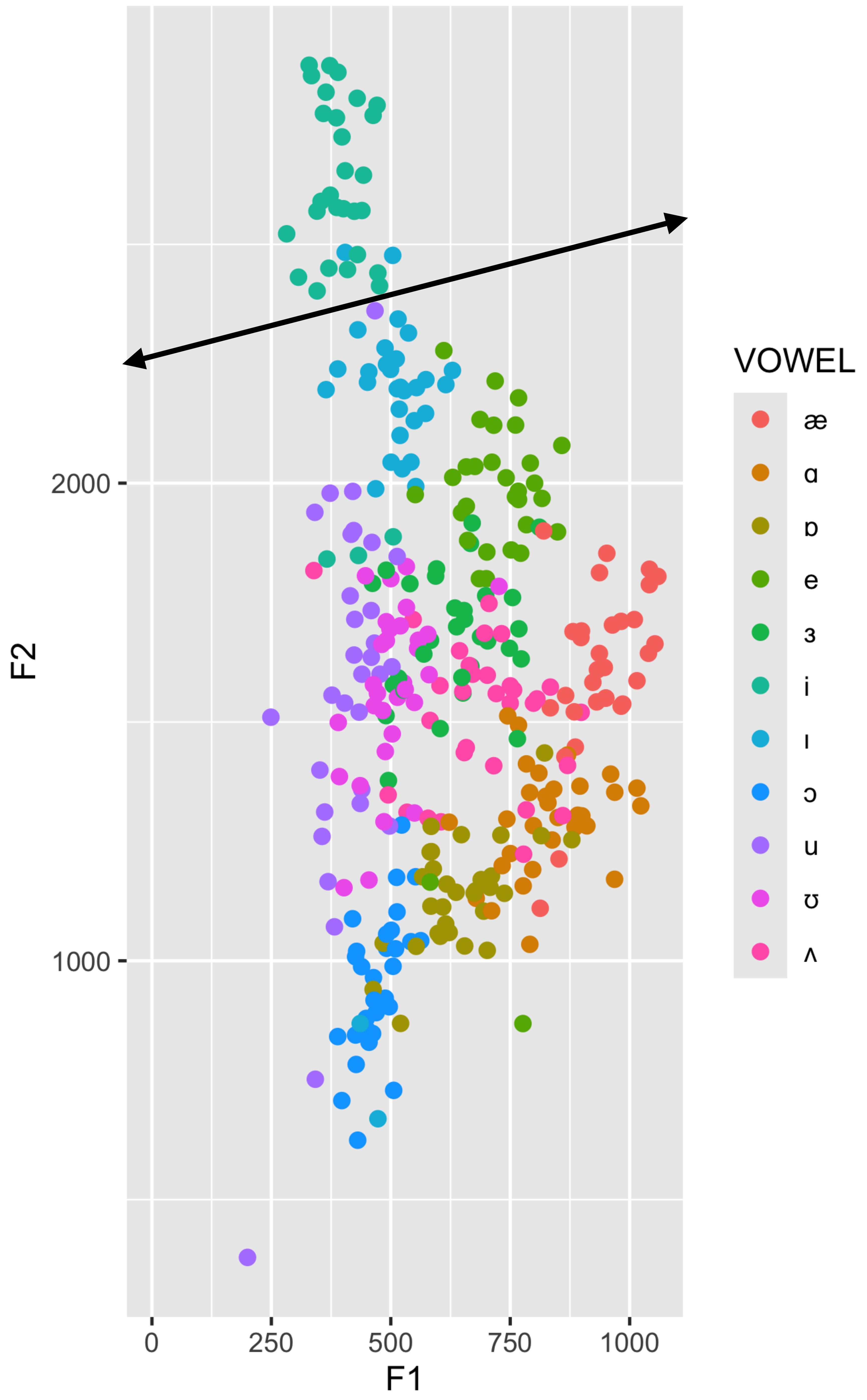

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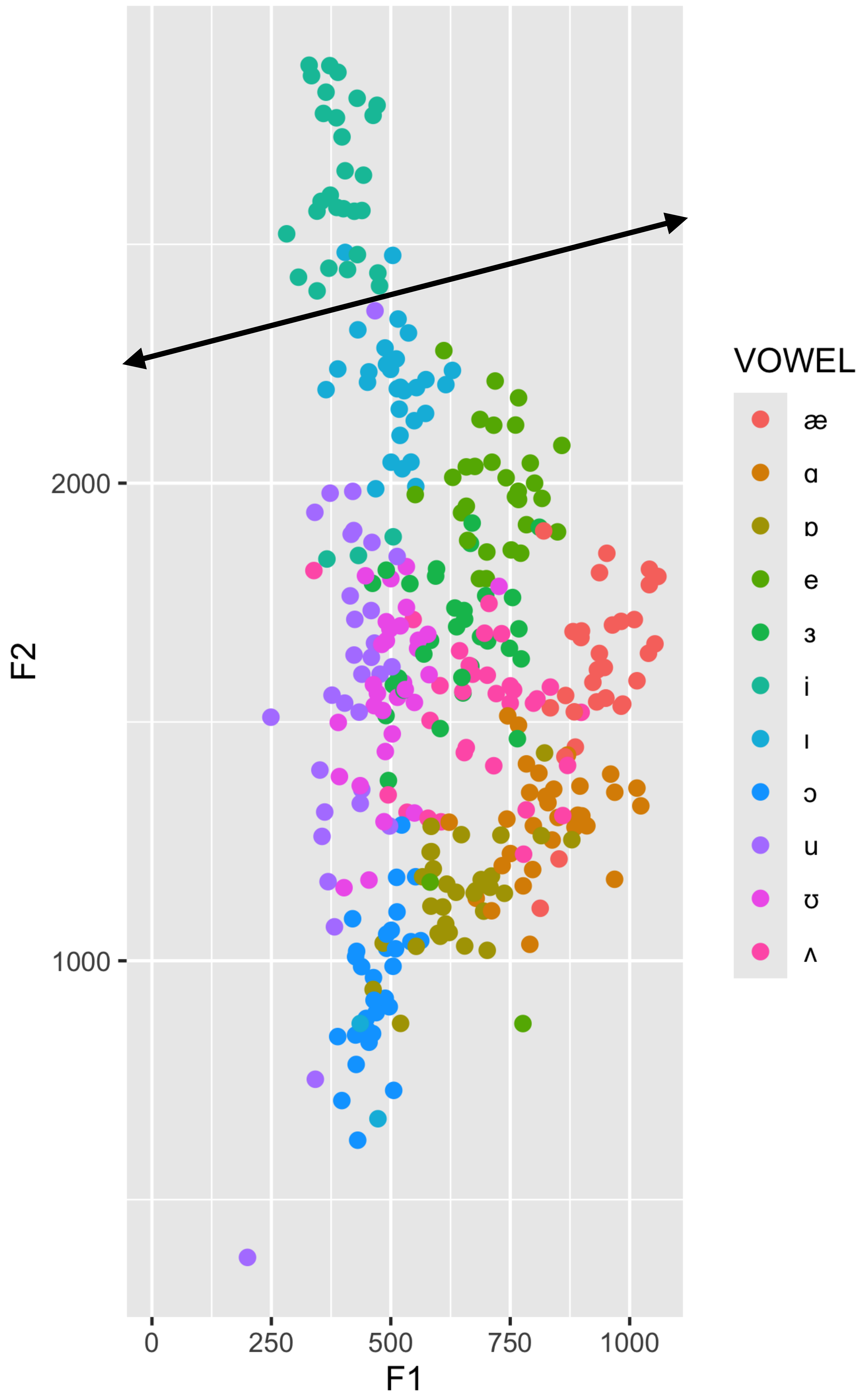

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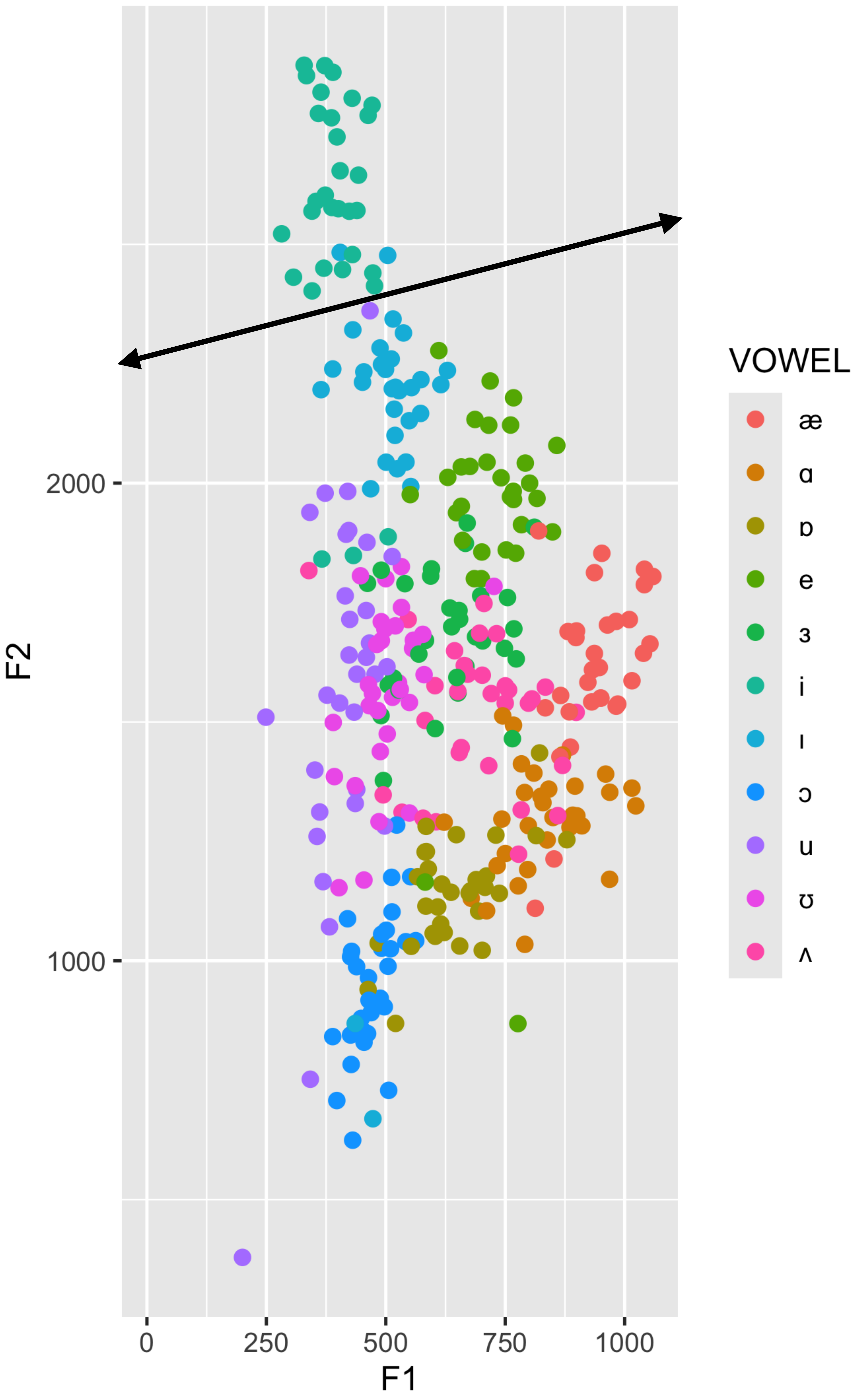

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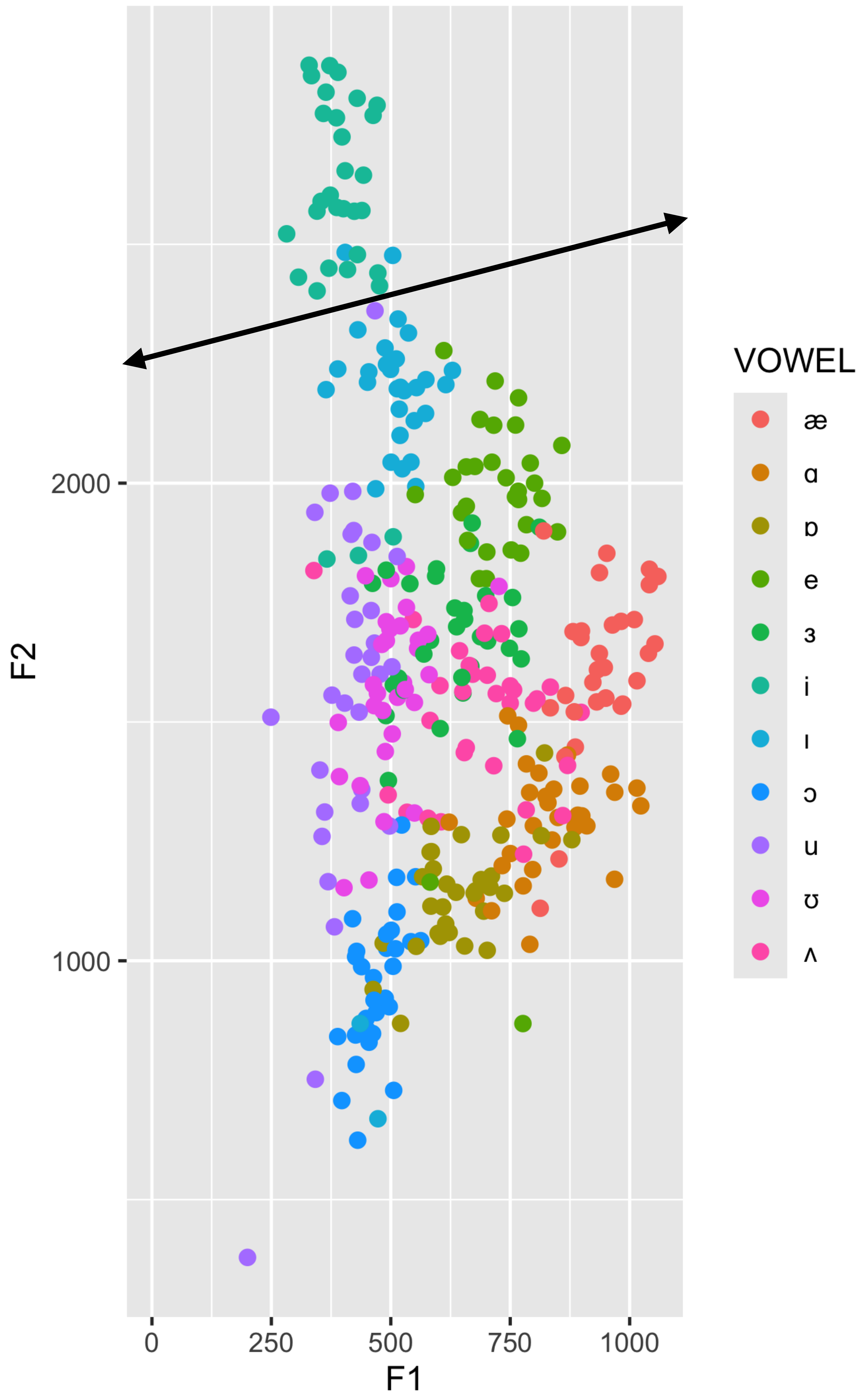

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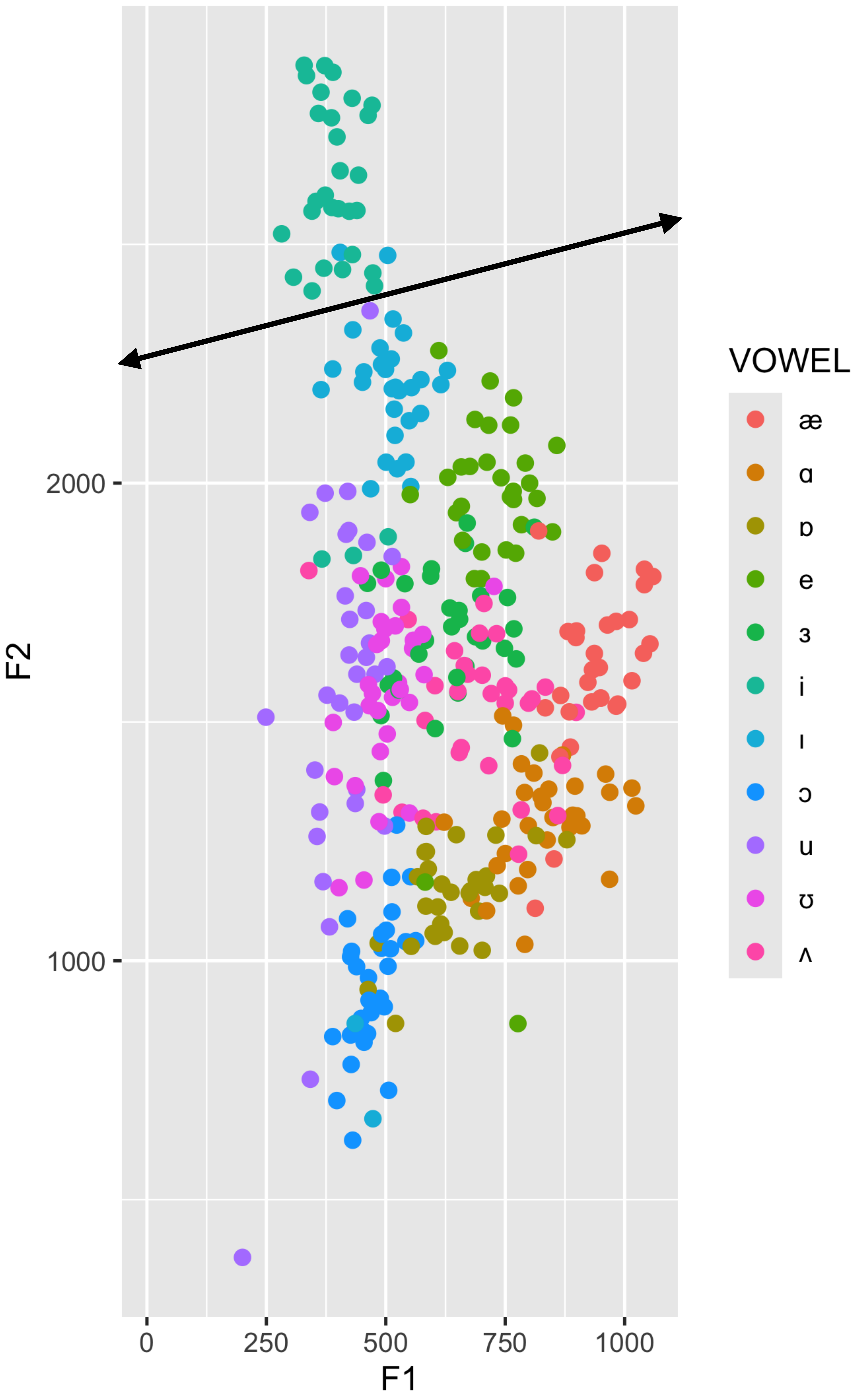

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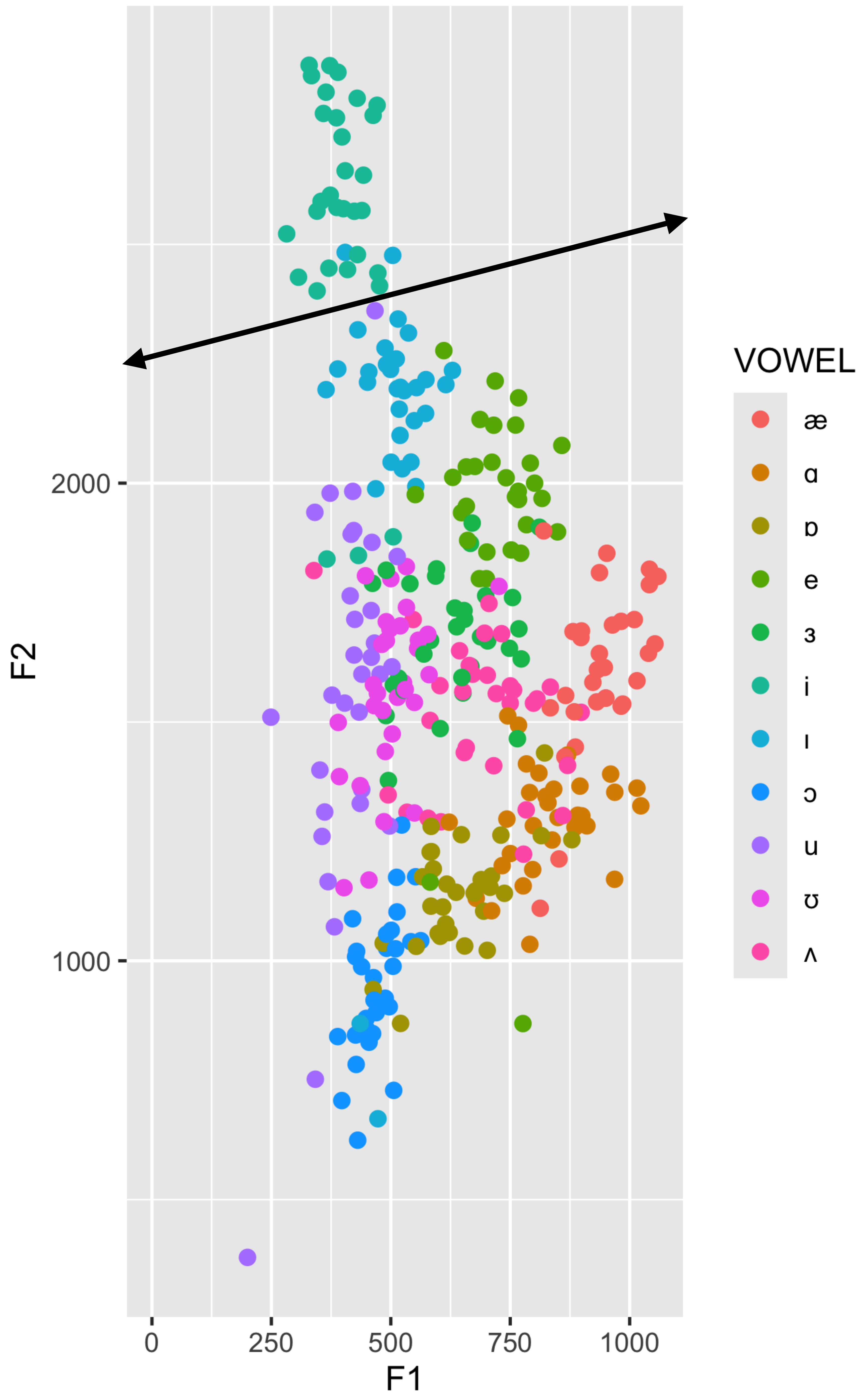

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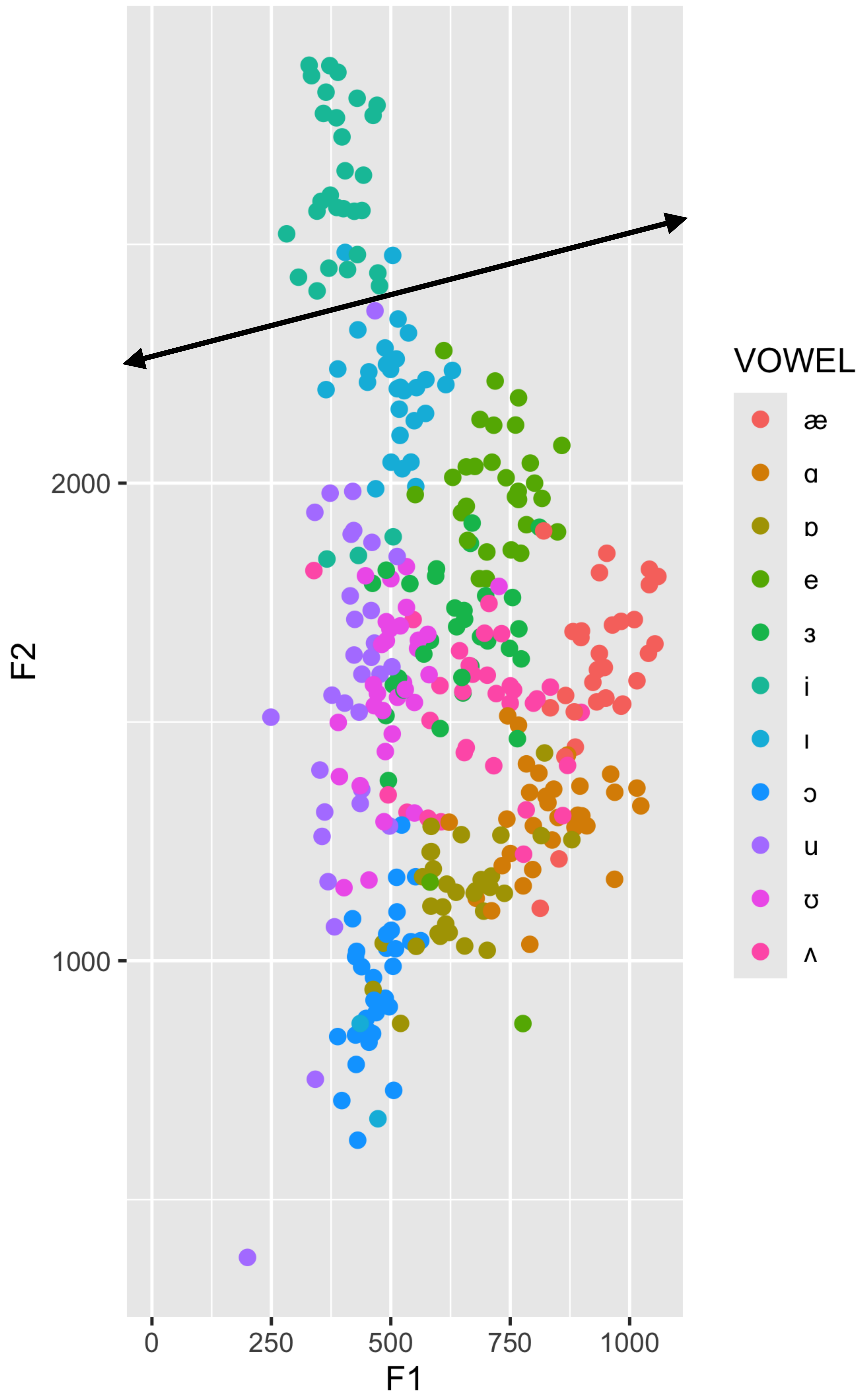

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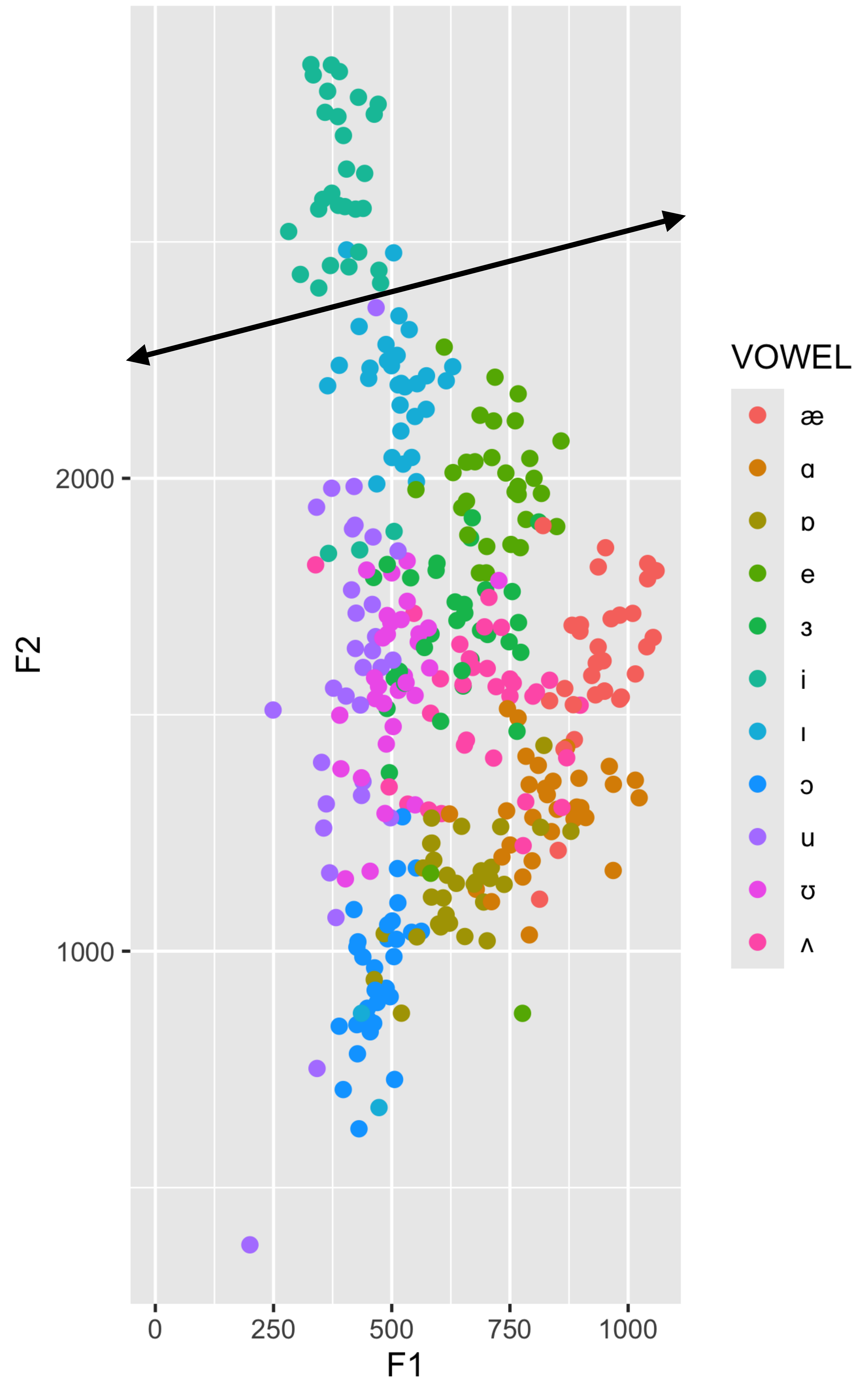

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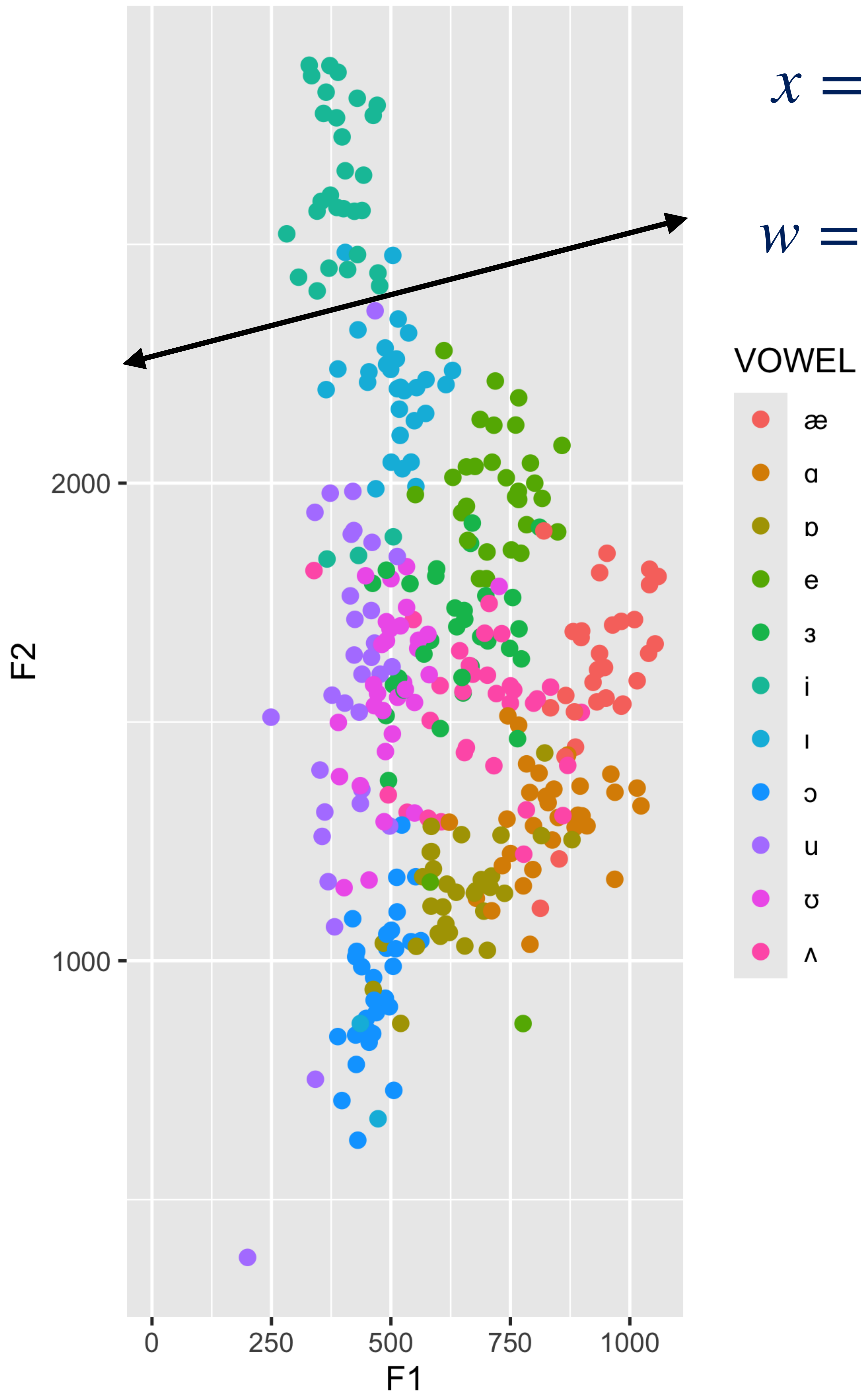

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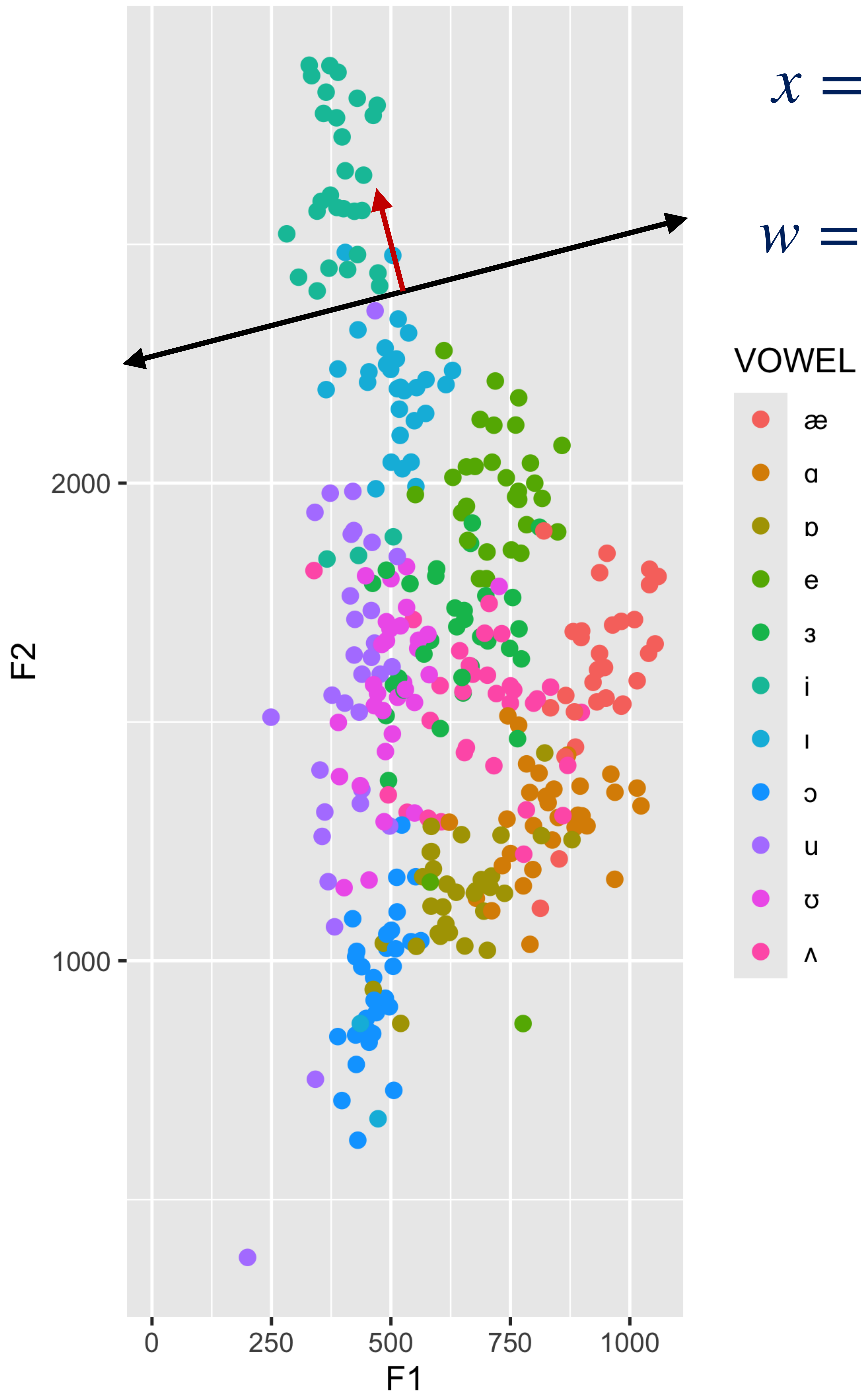

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Logical Functions

p	q	p∧q	p∨q
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	1

Logical Functions

- Logical (or "Boolean") functions give **truth values** based compositionally on the truth value of their inputs

p	q	$p \wedge q$	$p \vee q$
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1	0	0	1
0	1	0	1
1	1	1	1

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- This **truth table** gives the values of **logical AND (∧)** and **logical OR (∨)** based on some input variables p and q

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- **"Boolean logic"** is pervasive in many fields such as **Semantics, Computer Science, and Electrical Engineering**

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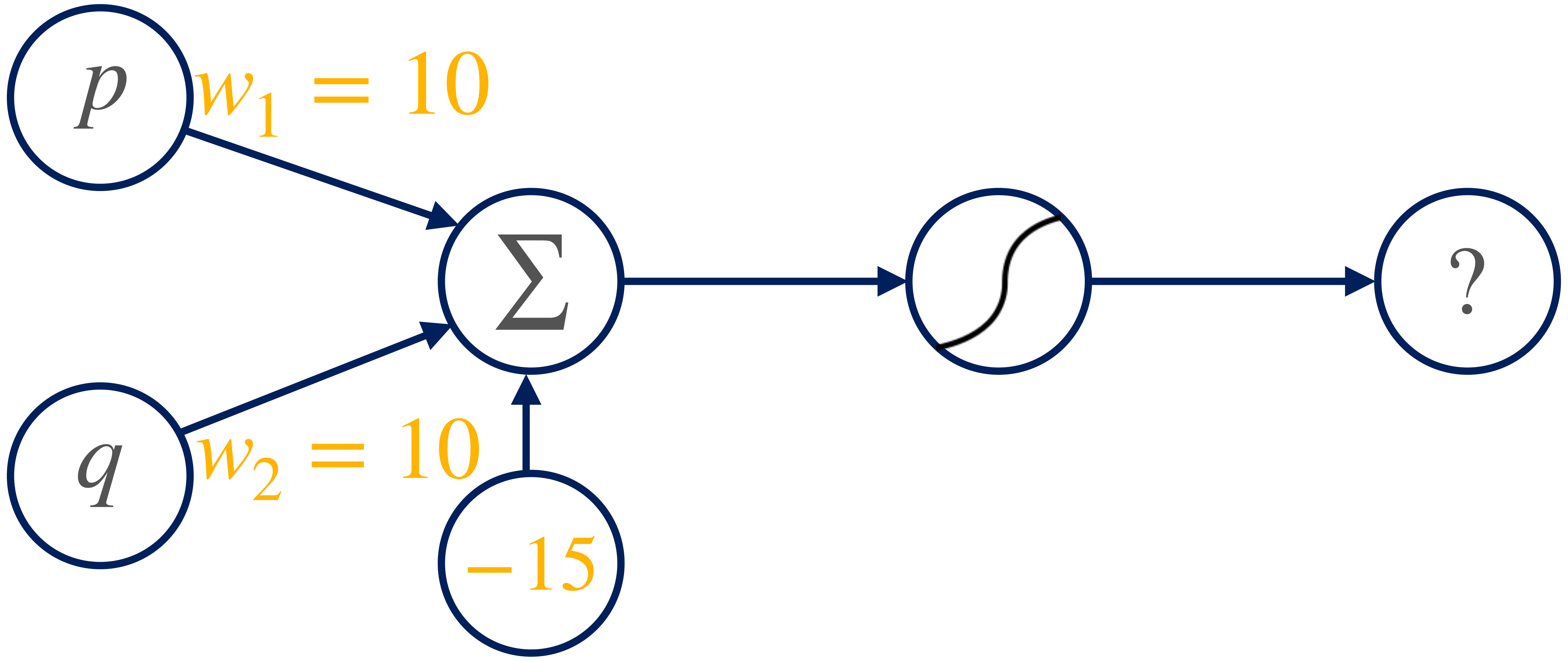

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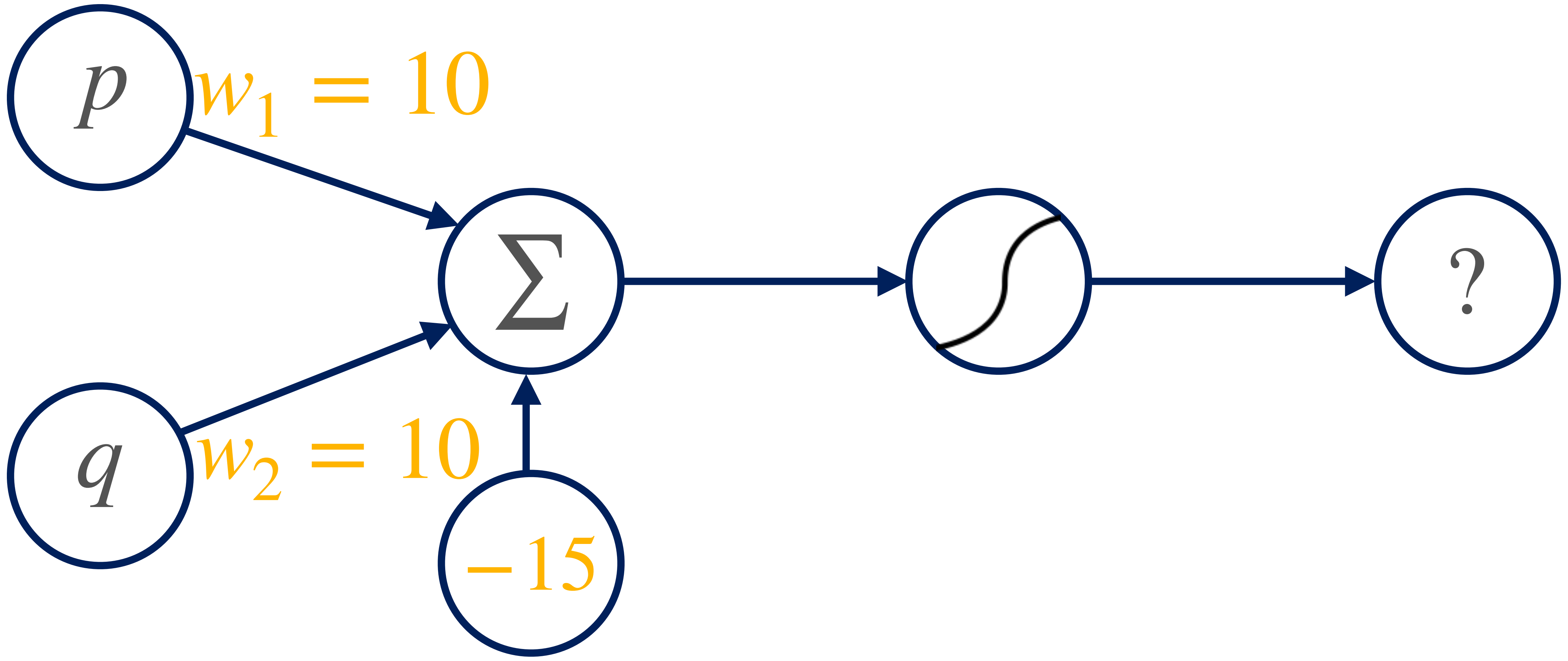

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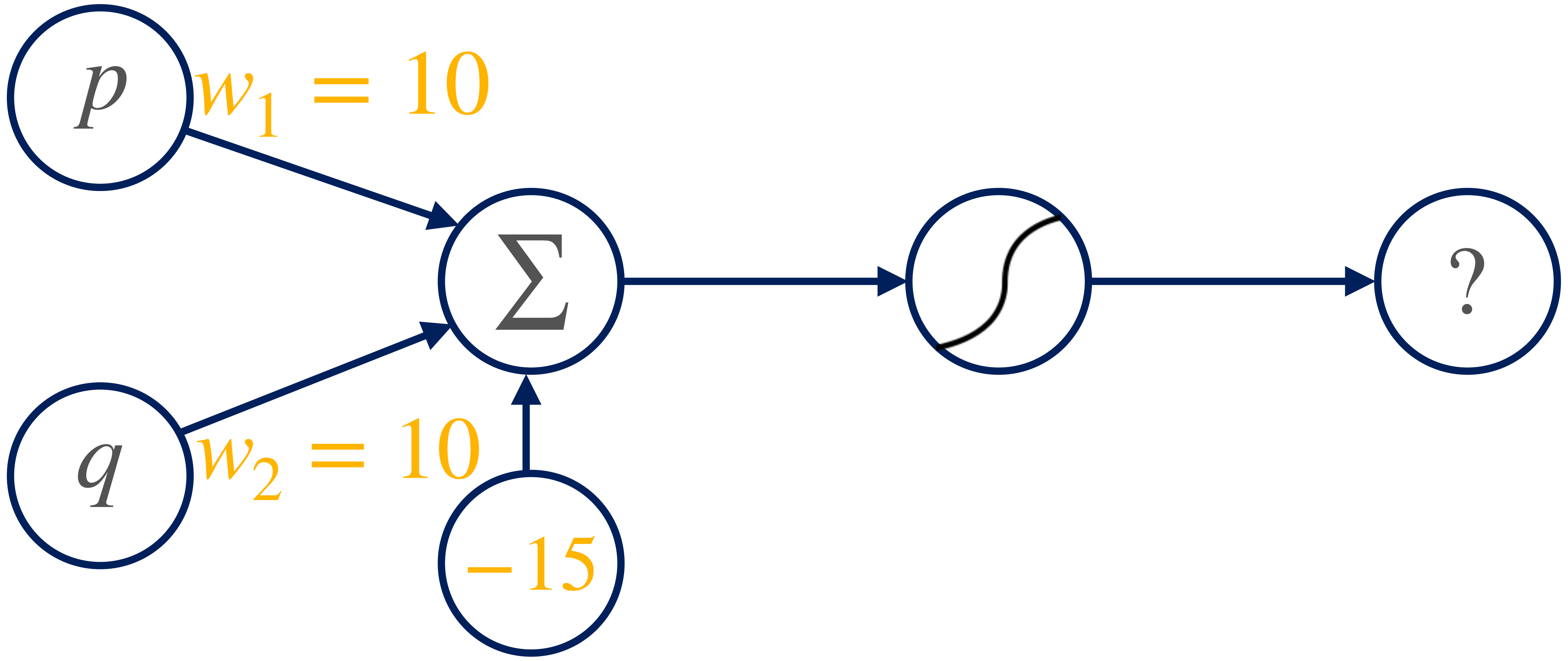
- When is Perceptron output **close to 1.0?** What is $wx + b$?



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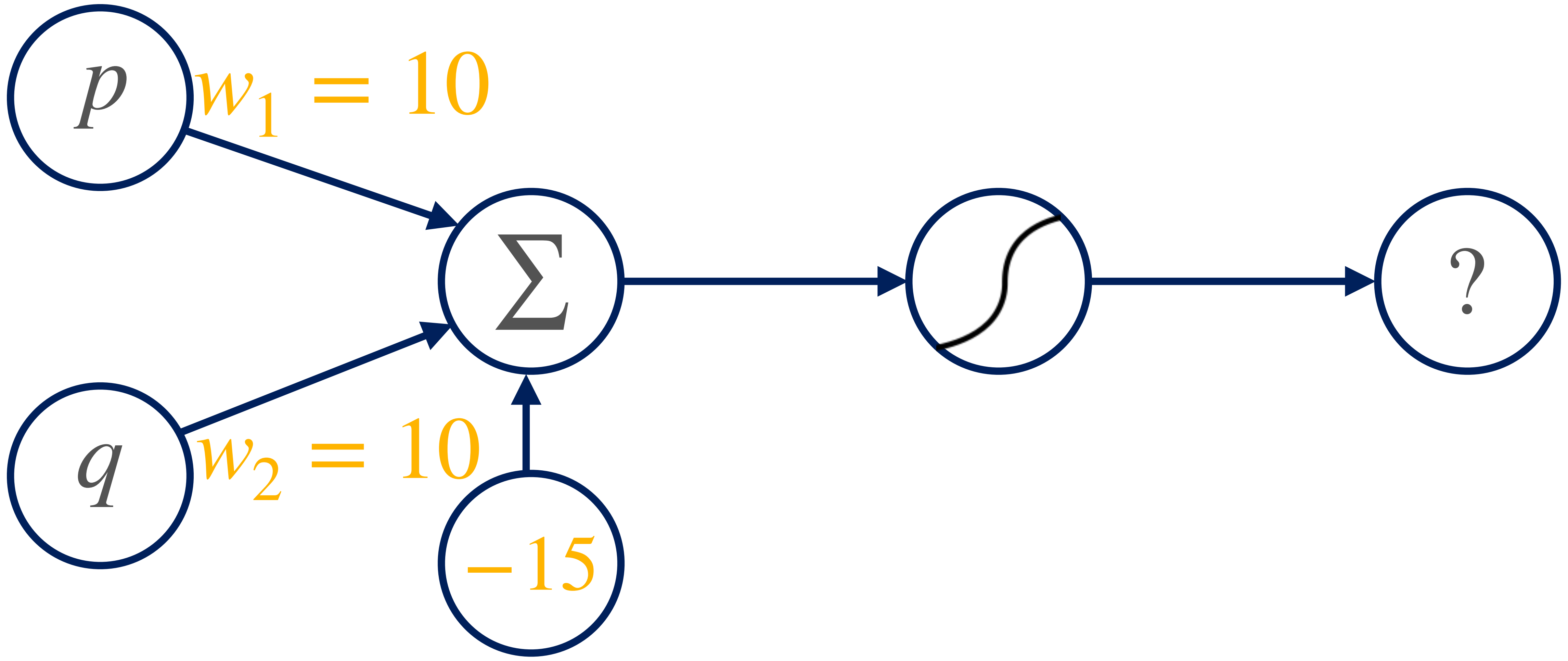

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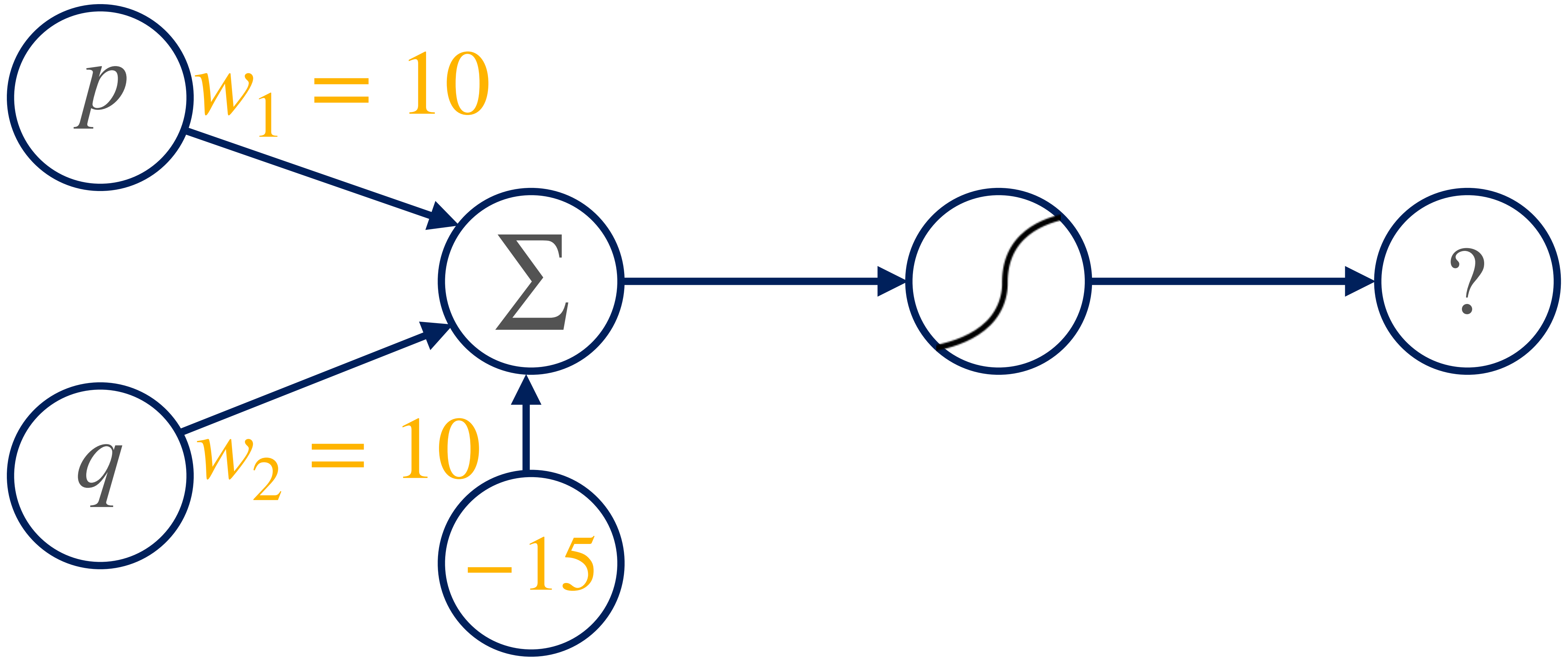

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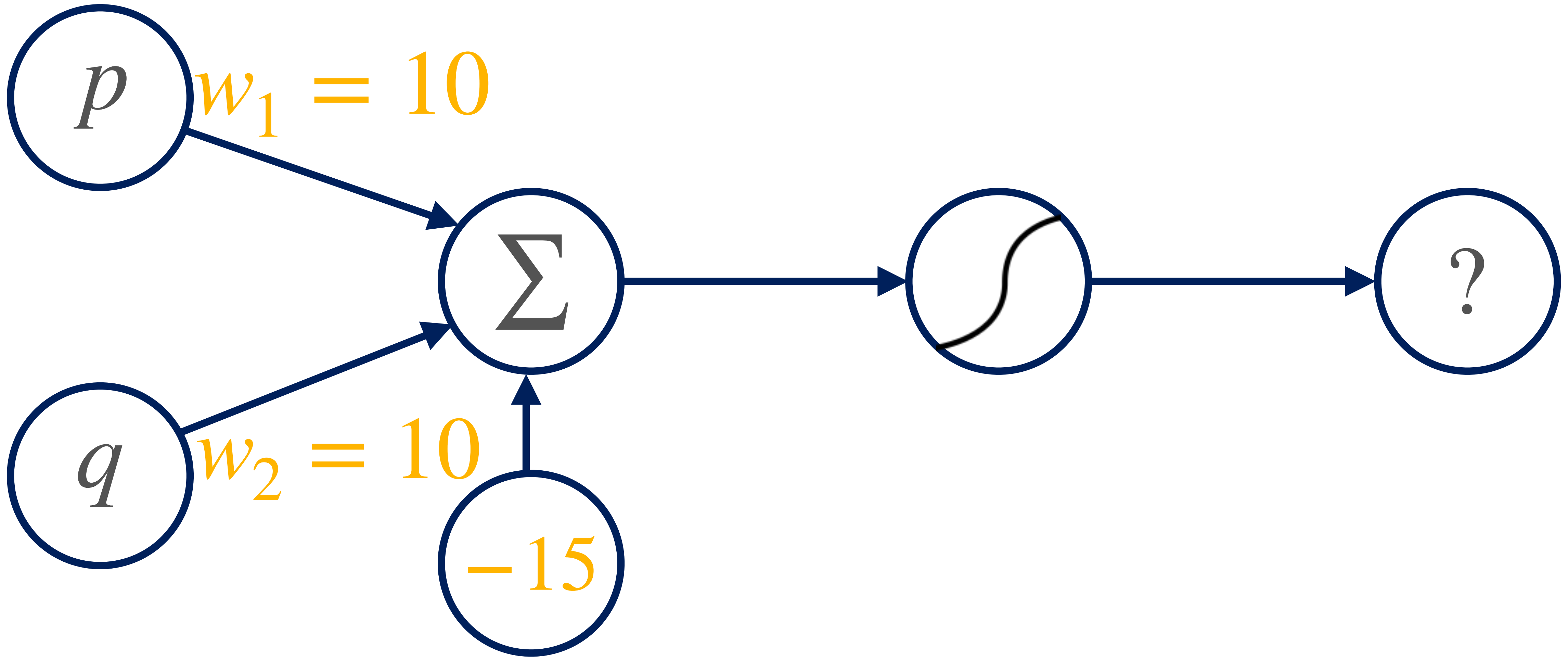
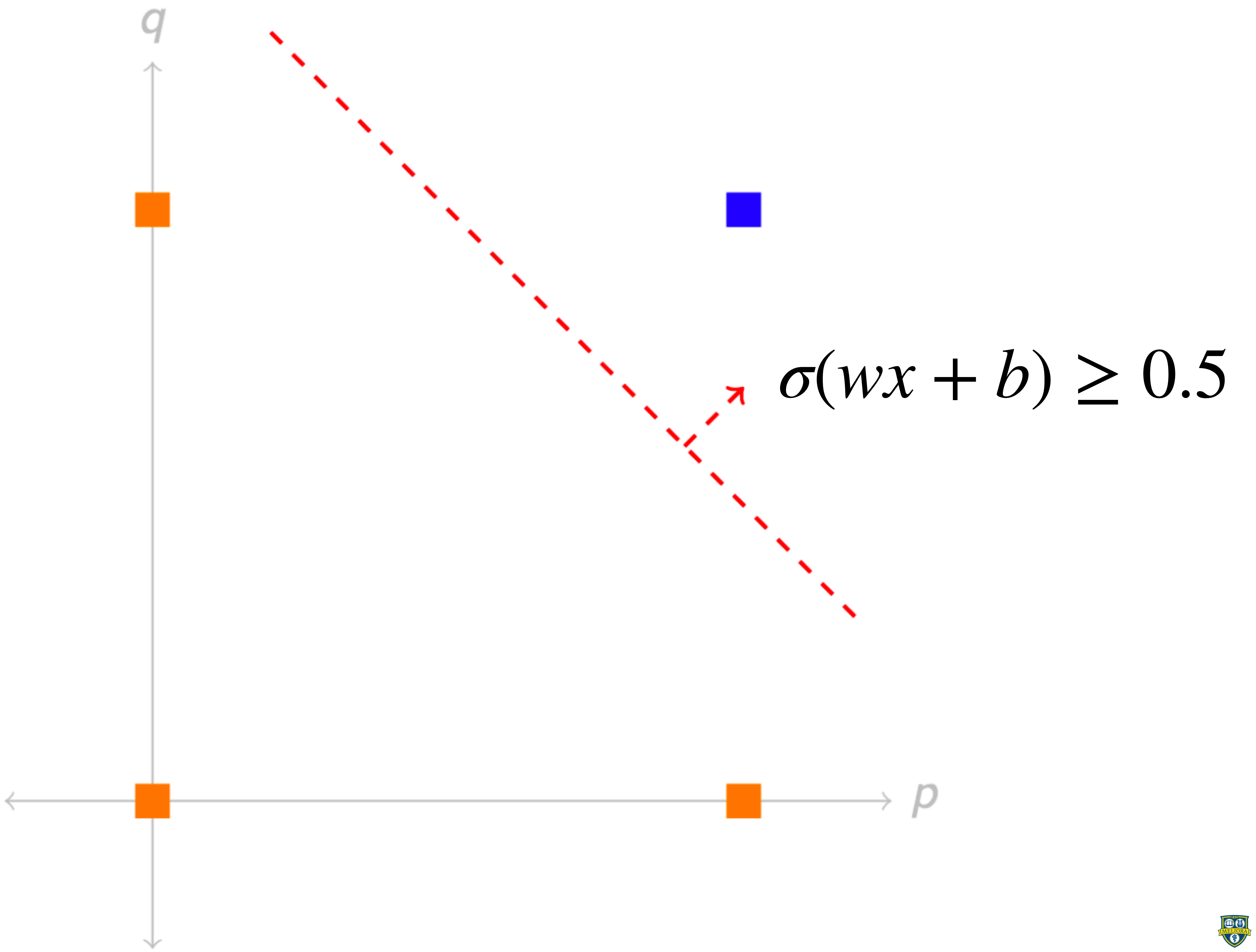
- When is the output **close to 0.0?** What is $wx + b$?
  - When $wx + b$ is **negative**, i.e. **either p or q is 0**

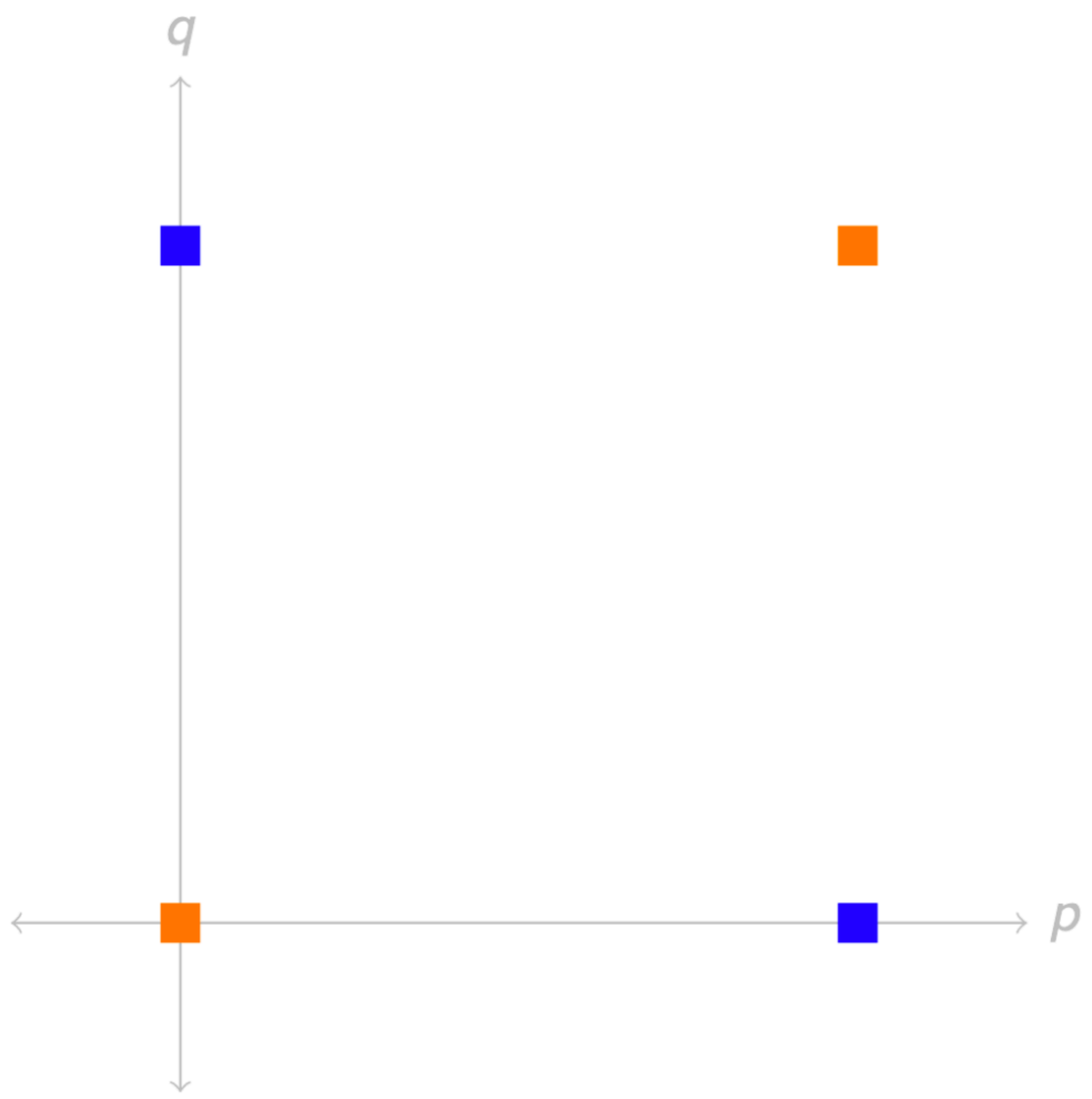


AND Linear Separation



XOR Function

p	q	p XOR q
0	0	0
1	0	1
0	1	1
1	1	0



XOR Function

- The XOR function is True when **either** p or q are True **but not both**

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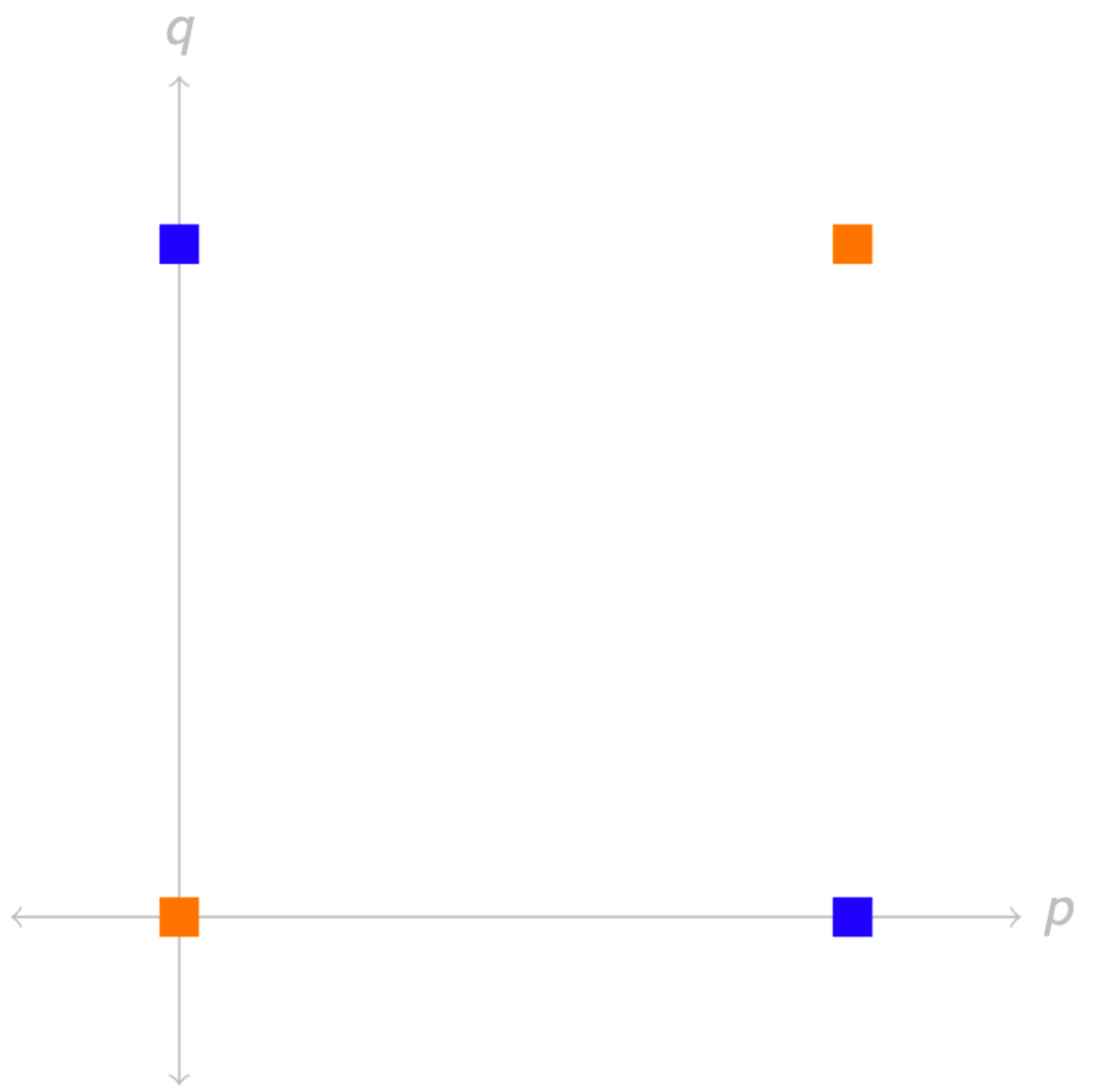

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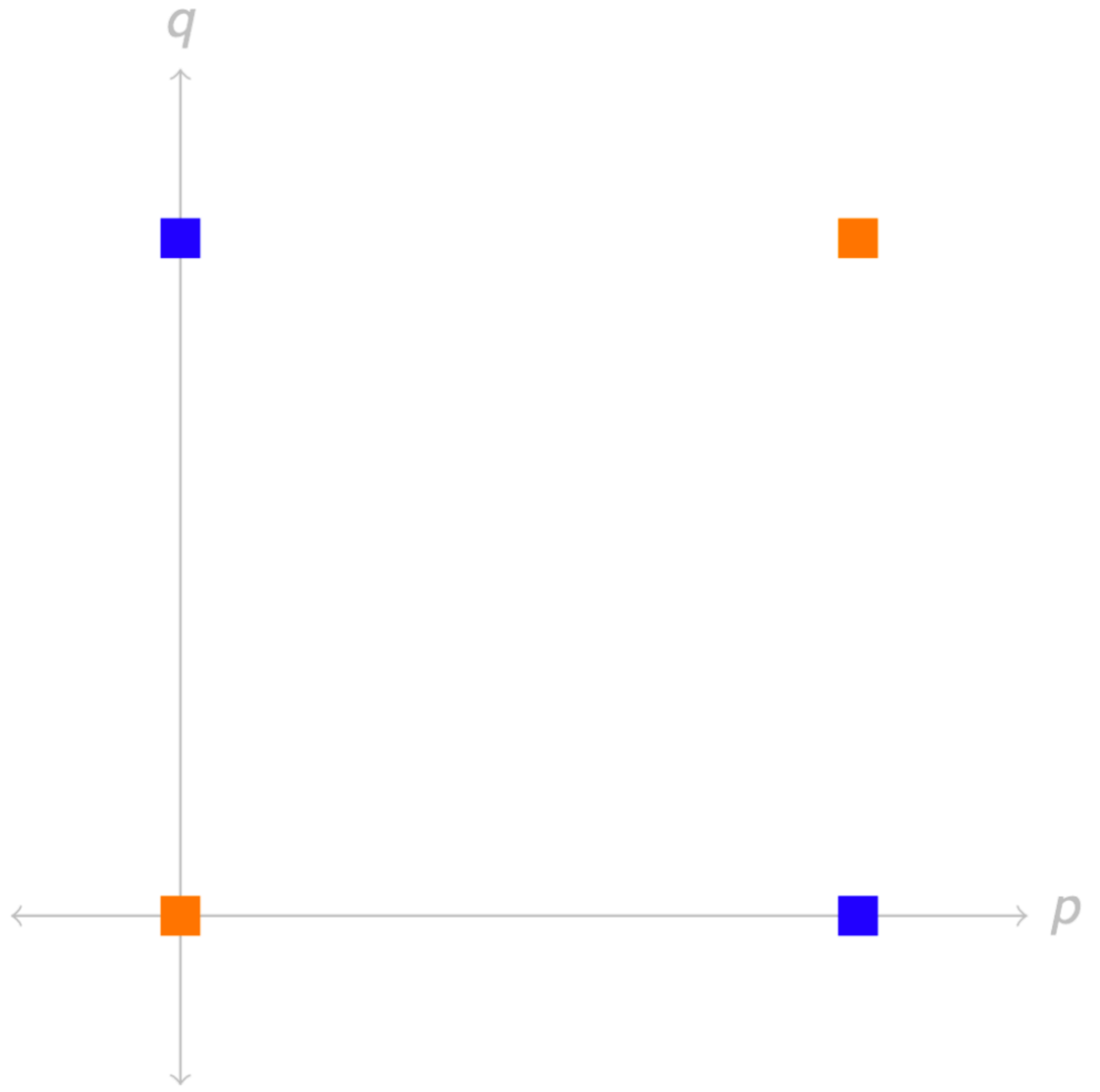

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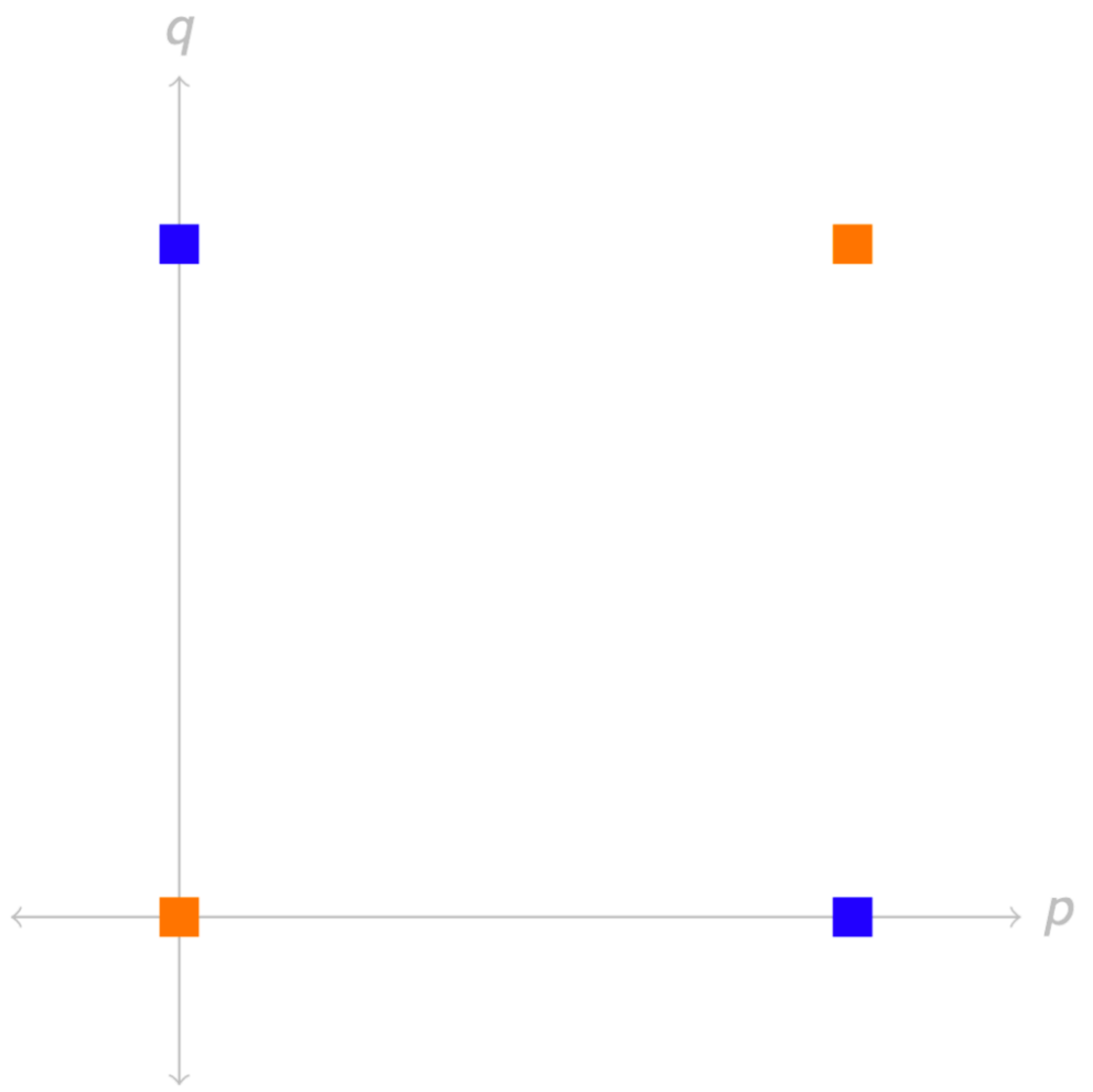

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  - How do we model functions like this? More in a later lecture

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0	1	1
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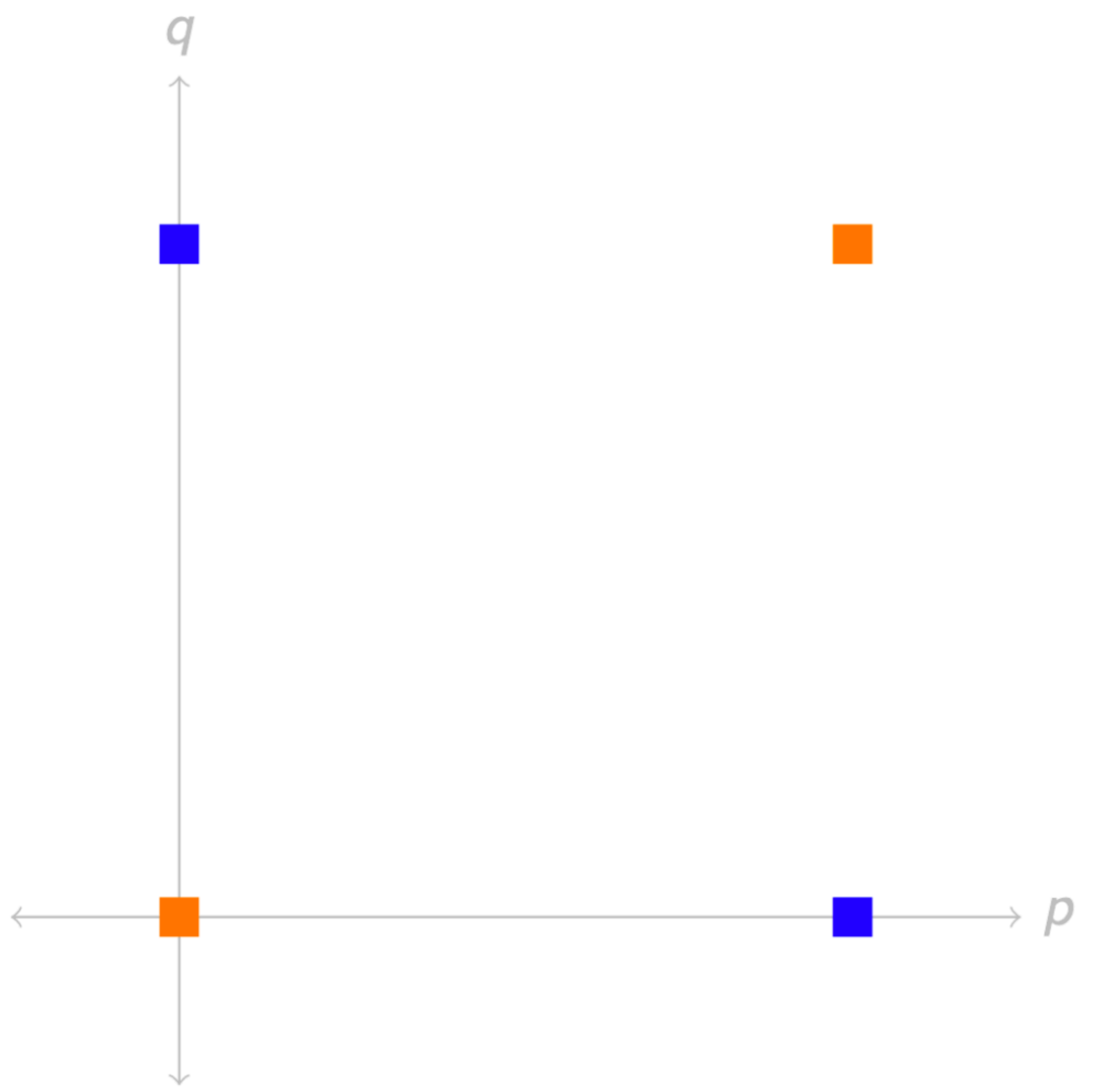

Last thoughts on Perceptrons

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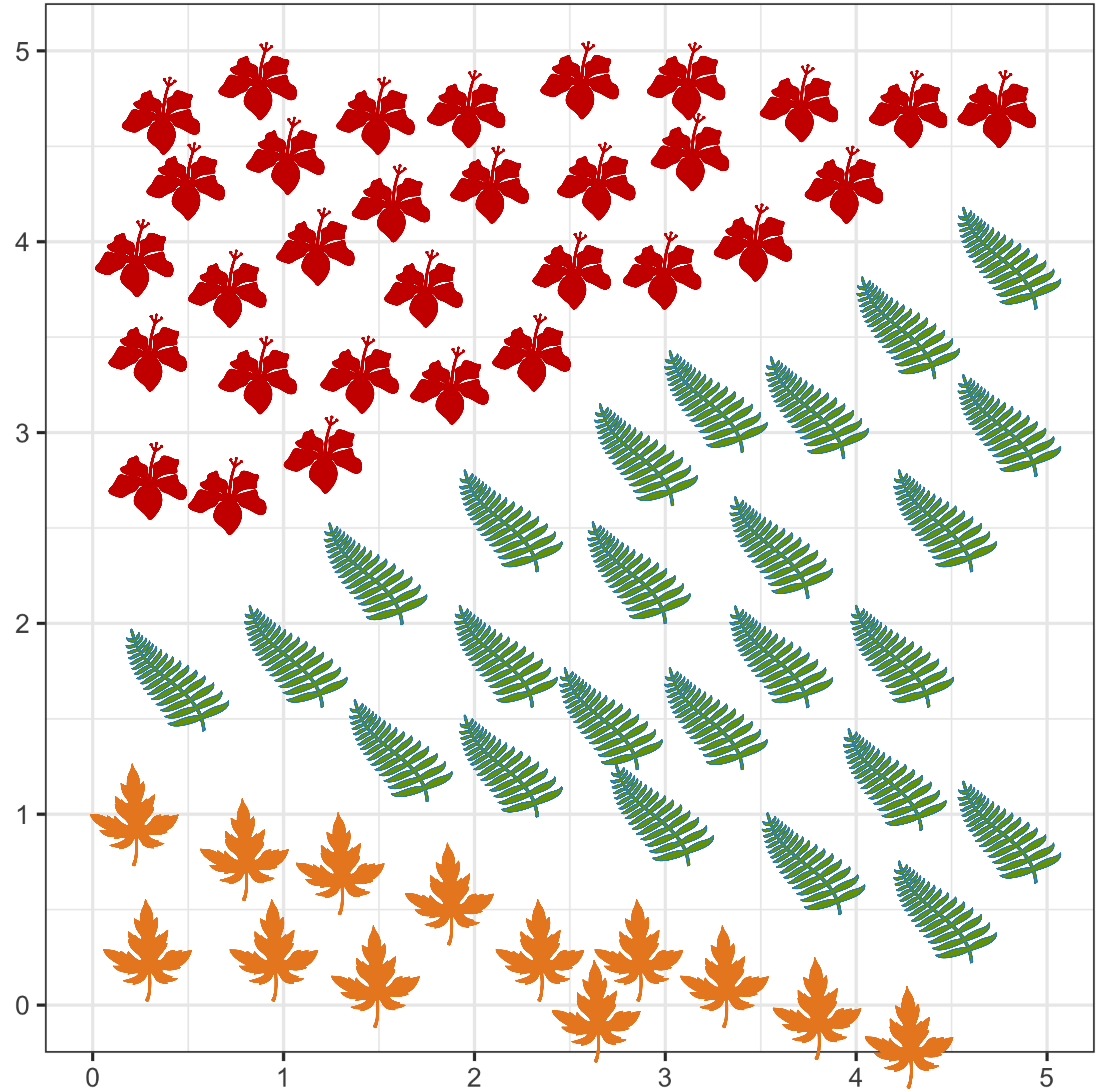
  - e.g. Artificial Neuron, **Logistic Regression**, Maximum Entropy (**MaxEnt**)
  - These boil down to the **same model**! (with some difference in training algorithm)

Last thoughts on Perceptrons

- The Perceptron model goes by **many different names**
  - e.g. Artificial Neuron, **Logistic Regression**, Maximum Entropy (**MaxEnt**)
  - These boil down to the **same model**! (with some difference in training algorithm)
- Important points:
  - A Perceptron encodes a **linear separation** in n-dimensional space
  - Like a human neuron, the Perceptron "fires" when the **weighted inputs exceed some threshold**
  - The **weights** need to be **learned from some algorithm** (more next time!)

Practice Example

How do we separate **both** the flowers and the maples from the ferns?



Quiz next time!

- Covering pre-requisites from first-semester Calculus
  - Applying rules to **take the derivative of a function**
    - I will give you a **cheat-sheet** with the rules
  - Extra focus on **how to apply the Chain Rule**
  - Understanding **critical points** of a function/derivative ($f'(x) = 0$)
  - Be able to **sketch a tangent line** at a point on a curve
  - Understanding of **what a function derivative is**
- **No need** for trigonometric functions (cos, sin, tan) or limits right now

Derivatives Cheat-sheet

$$\frac{d}{dx} x^n + c = nx^{n-1}$$

$$\frac{d}{dx} f(x) \pm g(x) = f'(x) \pm g'(x)$$

$$\frac{d}{dx} c \cdot f(x) = c \cdot f'(x)$$

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$\frac{d}{dx} ln(x) = \frac{1}{x}$$

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$